Multi agent simulation of pedestrian behavior in closed spatial environments

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Abstract – **Agent-based simulations show their potential in many context of transport management in presence of unusual demand, such as airport passenger terminals, railway stations, urban pedestrian areas, public buildings, street events or open space exhibitions, where management or control by related authorities and public safety are strongly affected by spatial geometry and crowd behavior. We illustrate these ideas with an example based on the simulation of people visiting and evacuating a museum, which offers an excellent test environment for simulating a collective behavior emerging from local movements in a closed space. The model we apply is developed within a programmable modeling environment, NetLogo, designed for simulating time-evolution of complex systems. We verify the existing emergency plan for building evacuation, for different demand patterns such as visiting group size and inter-arrival times, and we compare it with alternative evacuation strategies looking for the optimal one. In this respect, we further demonstrate the effectiveness of agent-based simulations in finding emergent results difficult to be predicted.**

Keywords: Intelligent agents, Pedestrian modelling, Crowd safety, Simulation, Spatial interaction, NetLogo

I. INTRODUCTION AND LITERATURE REVIEW

The context of this paper is modeling pedestrian behavior. It is important to study pedestrian behavior for several reasons: walking, the most sustainable form of transport, involves 75% of all trips under a kilometer, transport interchange facilities, commercial centers, public spaces. However today there is still a disproportionate investment on research into motorized travel and we are not able to easily answer questions on how many people will use a new pedestrian facility or how will improvements to non-motorized travel conditions affect motor vehicle use. Also Urban Planning, Crime Prevention, Emergency, Disaster Planning and Epidemiology are fields which need improved methods for modeling pedestrian movements. Actually, walking is not a simple behavior suggested by the geometrical representation of a pedestrian trip as a line connecting origin and a destination. Concepts such as crowding, personal space, territoriality, sensory overload, collision avoidance, navigation and orientation, environmental perception, evaluation and decision making, are all important ingredients which have to be taken into account when modeling pedestrian behavior [1]. To analyze such a behavior at the very fine scale, while pedestrians move along the streets, in open spaces or inside a building, computational tools become essential. In transport engineering, models representing pedestrian flows can be roughly separated in analytical models and micro-simulations. The first ones represents pedestrian flows through mathematical models and provide an assessment of the average pedestrian flow along a street, the second ones simulate the movement of each single pedestrian following a set of pre-determined rules of behavior. Analytical models include “before and after” methods, regression analysis models [2], analogies with fluids, gas kinetics and other physical flow systems [3], application of gravity models [4], entropy maximization [5], dynamic network analysis with flow models calibrated on the basis of data collected [6], discrete choice model to predict pedestrians’ route choice [7], stochastic queuing and Markovian models [8]. The limit of these models is that they obviously cannot take into account some peculiar aspects of pedestrian (human) behavior. Actually, in closed spatial environments and in presence of unusual demand flows, simulations require the ability to model the local dynamics of individual decision making, which is strongly affected by the geometry, randomness, social preferences, local and collective behavior of other individuals. Micro-simulations have the potential to overcome the major limitations of the previously discussed models by incorporating a variety of rules to simulate movements rather than being based on rigorous assumptions and complex theory. They have the advantage of being applicable to a greater variety of situations once the appropriate calibration has been conducted. A very interesting approach is, for example, the one by Helbing and Molnár [9], based on the concept of “social force”, a measure for the internal motivations of the individuals to perform certain actions (movements) and its influence on people’s dynamic variables (velocity, acceleration, distance). The rapid increase in computational speeds and rich data sets has led to the increased accuracy of applications of this modeling technique. In particular, cellular automata and intelligent agents simulations have had a huge development in recent years. The first approach models the behaviour of individual pedestrians by means of a limited set of rules describing its features [10], while the second one allows to treat pedestrians as fully autonomous entities with cognitive and often learning capabilities [11].

In this paper we perform an agent-based simulation of people visiting and evacuating a museum, located inside the Castello Ursino in Catania (Italy), which offers an excellent test environment for showing the potentiality of this technique. The building is modeled in 2D with its real geometry; it has only one entrance but many rooms and corridors containing paintings, sculptures and other works of art which contribute to offer several destinations for visitors that can be differentiated in size of the visiting groups and preference for art lovers. The interval of time during which each visitor stands in front of an artwork is affected by the expected interest (which can be different for different visitors) and by the number of people which crowd the room, a situation quite similar to the one of people crowding an airport terminal commercial area before departure. The aim of the paper is to simulate the visitors' behavior during the normal fruition regime and to observe their reaction to a sudden alarm situation in correspondence of different arrangements of the emergency exits and different evacuation strategies. More in general, we want to show how an agent-based simulation of pedestrian walking dynamics in a given bounded environment can be important in exploring rising crowd behavior which can affect, in turn, the level of service and the safety of a building [12].

II. METHODOLOGY AND CASE STUDY

A. *Agent-based simulations: the NetLogo platform*

An agent-based simulation is a computer technique simulating a system whose main components are 'agents', i.e. single mobile entities which are capable of autonomous actions in order to meet their fixed objectives. Agent-based simulations can be implemented over software platforms able to simulate a virtual spatial world where agents interact each other and with the spatial environment, according with a control system which defines their behavior at the micro-level, that is their reaction to external outputs. Although each agent is modeled by variables and algorithms, this does not imply a stiff resulting behavior. On the contrary, agents are capable of reactive, proactive and social behavior: reactive means they can perceive and respond to changes in their environment, proactive means they can take the initiative to achieve their goals, and social means they can interact with other agents to satisfy their objectives. Therefore, the emerging collective behavioral patterns at the macro-level can be regarded as the result of a trade-off between competitive and cooperative individual choices. The typical case is when local pedestrian movements towards some goal can lead to undesired crowded situations while the tendency to follow what others are doing (herding effect) can favor congestion and panic. In other words, these simulation techniques are able to capture and explore rising crowd behaviors which cannot be described as simple aggregations of individual movements and that often are very far from intuition and hardly foreseeable by experience and common sense [13,14,15].

There are different integrated development platforms to assist agent-based simulations. A common feature of these platforms is to allow model development, code scripting and simulation running, all in the same graphical interface. Among several of them we chose NetLogo [16], which is a simple but powerful multi-agent simulation environment written in Java but fully programmable in an owner powerful meta-language of higher level. NetLogo is freeware, direct and friendly, and allows the designer to focus directly on the agents' properties rather than to be drowned in complex code. Actually, modelers can give instructions to hundreds or thousands of individuals, all operating independently and programmed in terms of how they interact with each other and with the environment. In order to make this easier, Netlogo allows to create an interface for each model with commands in the form of procedures, activated by buttons on the interface, or entered directly in the command console. It also provides many built-in tools to modify simulation parameters at runtime, including sliders, buttons and drop-down menus and allows output in the form of graphs, variable monitors or external files. Due to its features, NetLogo is particularly well suited for modeling complex systems developing over time, being time measured in discrete arbitrary units (ticks) which can be rescaled by the user and converted in real time ones (seconds, minutes, etc). The spatial environment in NetLogo is represented by a two dimensional grid (World) of discrete cells, called 'patches', whose topology can be chosen among square/rectangular (with closed boundary conditions), cylindric (with open boundary conditions on one direction) or toroidal (with open boundary conditions on both directions). Unlike pixels in a picture, patches are active motionless elements, since they can possess several features (coordinates, colors and other user-variables) and can be able to interact with moving agents which pass over them. We will give other details on the Netlogo environment in the next section. Now let us introduce the case study of this paper.

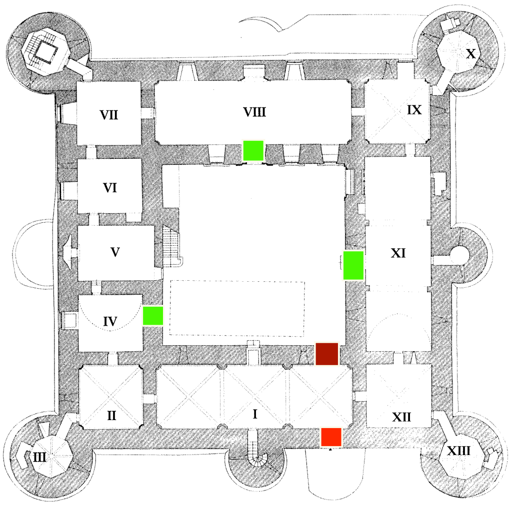

Figure 1. Real Planimetry of Castello Ursino

B. *The Castello Ursino Museum*

Using Netlogo, we apply the agent-based approach to study pedestrian behavior inside a complex old building, the Castello Ursino, built by order of Federico II between 1239 and 1250 as part of a much larger scheme of fortresses spread out over the Sicilian territory. The layout is shown in Fig.1: the planimetry is square, with four buildings placed around a central courtyard and the corners are buttressed by large cylindrical towers. Since 1934 the castle has become the permanent headquarters of the Civil Museum of Catania, with highly interesting archeological and artistic artefacts. Castello Ursino building has four floors, but at the moment only the ground floor, represented in Fig.1, is accessible to the visitors. From the only Entrance/Exit door (bright red square) users walk counterclockwise and visit the sequence of rooms in a decreasing number order: XII, XI, … , II, I. Three emergency

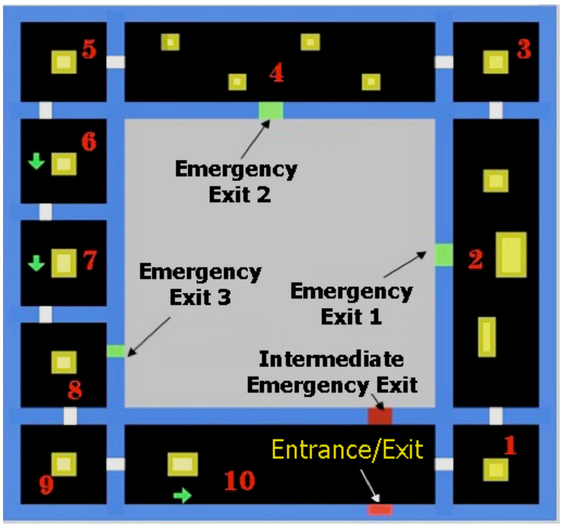

Figure 2. Planimetry of Castello Ursino, realized on the NetLogo spatial grid (World) . Pixels of different color correspond to patches of the grid with different functions (see text).

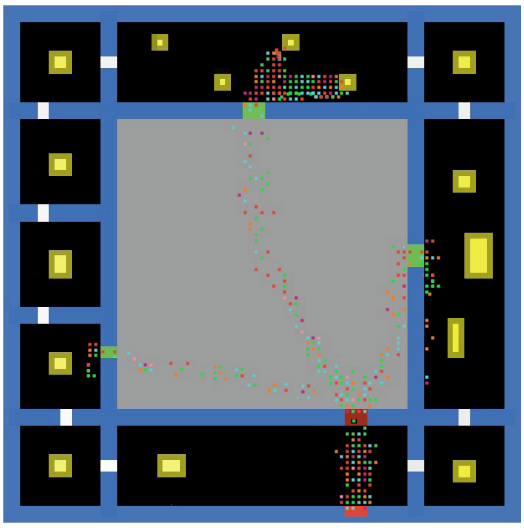

Figure 3. An example of evacuation through the courtyard in the existing configuration of three emergency exits without emergency signs (see text).

exits (green squares), located respectively in rooms XI, VIII and IV, overlook the central courtyard, in which people should converge in an alarm situation (following the existing evacuation plan). Then, through an intermediate emergency exit (dark red square), they should flow in room I and definitively exit from the same Entrance/Exit door they came in.

Castello Ursino civic museum has been chosen as case study for different reasons: (i) it has been previously studied by means of NetLogo agent-based simulations by one of the author for its socio cultural aspects (carrying capacity of the building compared with the level of satisfaction of the visitors [17]); (ii) As most of the building belonging to historical heritage, there are some concerns about safety and emergency management; (iii) The building provides a good study case for simulating local movement of visitors in a situation of normal fruition, as there is only one main Entrance/Exit door but many rooms which contain different paintings and artworks, thus providing different levels of interest to visitors; (iv) The existence of the three emergency exits overlooking the internal courtyard and of the intermediate emergency exit before the last external Entrance/Exit door, makes possible to consider several different configurations in simulating the alarm/evacuation regime. All these factors made Castello Ursino an optimal environment for testing the ability of agent-based simulations to explore different pedestrian strategies for evacuating a public building. In the next section we show how to implement the Castello Ursino map and the corresponding visitors' dynamics into the NetLogo platform, and we report the simulations results.

III. IMPLEMENTATION OF THE AGENT-BASED MODEL AND SIMULATION RESULTS

A. Normal fruition dynamics

In Fig.2 the planimetry of the ground floor of Castello Ursino museum has been reproduced within the NetLogo interface by using a square grid with closed boundary conditions (the towers in the four corners have been neglected for simplicity). The grain of resolution is a patch, corresponding to a square of 60x60 cm^2 and able to carry only one visitor at a time. All the sizes of rooms, doors and walls were drawn preserving the scale of their real counterparts (in this scale, for example, the grid side is of 90 patches, corresponding to a total length of the Castle side of 54 meters). Each patch's color indicates a different "physical property" of the environmental object which agents recognize and with which they properly interact: external walls are blue, internal doors white, emergency exits green, artworks light yellow, fruition spaces around the artworks dark yellow and free space black (in this paper we chose a likely arrangement of artworks, with a total fruition area of 120 m^2, over about 2000 m^2 of total floor area). Besides there are green arrows in some rooms which represents emergency signs showing the closer exit direction. In the normal fruition situation, visitors (agents) access the red main entrance of the museum in groups of randomly selected sizes S (between 1 and a maximum value S_{MAX} fixed by the user) separated by random intervals of time uniformly distributed around the quantity ΔT (inter-arrival time) with a width of ±10 time units. Since we are mainly interested in a comparison among different evacuation plans and not to absolute values of time, we could keep NetLogo standard units (ticks) for evaluating time steps. Anyway, as agents move along one patch in one tick, considering the equivalence 1 tick = 1 second we would obtain an average velocity of 0.6 m/sec, which well approximates the normal pace (0.75 m/sec) and seems then appropriate for the fruition dynamics. Using this convention, in the following we will express time intervals in seconds, even if they could be conveniently rescaled when the evacuation dynamics will be considered.

Coming back to the description of normal fruition dynamics, visitors start their tour visiting each room in a fixed sequence, indicated in Fig.2 with increasing natural numbers inside the rooms. The motion of the visitors, though is

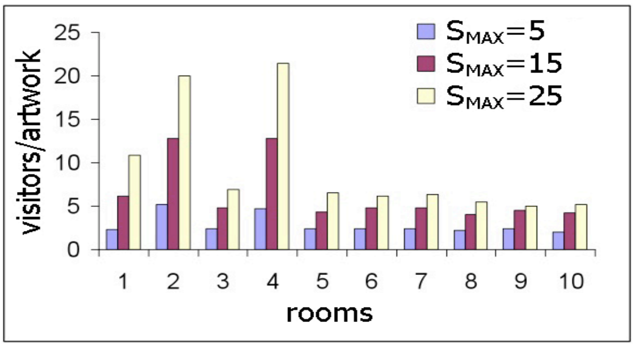

Figure 4. Normal fruition dynamics simulation: number of visitors per artwork for three different group sizes accessing the museum with a interval-arrival time ΔT=150.

constrained by the color of the patches, is not predetermined and follows a few intuitive rules. Each agent possesses a tunable radius of vision and is attracted by doors and artworks around him. At each time step, in absence of obstacles, each agent moves along one patch distance towards his target, i.e. a door or an artwork, which are dynamically stored in an individual list (memory) so that one visitor will not pass twice through the same door and will not attend twice the same artwork. In presence of obstacles, walls or other people, agents try to avoid them going around, until they reach the next door or the next fruition area near an artwork. For each artwork, a given agent has a random interest, i.e. the time that he is willing to spend enjoying that artwork, and a random patience, i.e. the time that he is willing to spend waiting for a place in the fruition area around that artwork. Both degree of interest and patience threshold are randomly selected within a fixed range. As a function of the patience, the interest and the time really spent for every artwork, it is possible to calculate a real-time quantity which expresses the average degree of satisfaction of all the agent visiting the museum (i.e. the global satisfaction index). Once the visit is over, visitors exit from the only external door, which is the same used to access the museum.

In a previous sociological study of Castello Ursino, by using a similar agent-based simulation performed with NetLogo, the carrying capacity of the building is investigated and compared with the level of satisfaction of the visitors [17]. It was found that the maximum number of visitors able to simultaneously visit the museum with a positive global satisfaction index was around 150. Moreover, he found that the average global sojourn time has a rapid increase with the number of visitors but, from 35 visitors onwards, it does not vary any more. In the next subsection we will set apart any consideration about the global satisfaction and we will focus on what happen when, after a transient regime of normal fruition of the museum, an alarm situation suddenly arises. In such a situation, of course, the behavior dynamics of the agents changes. Actually, as soon as an emergency alarm randomly goes off, each visitors checks if there is one of the three emergency exits overlooking the central courtyard within his radius of vision. If yes, he moves towards that direction, otherwise, if emergency signs are present and visible to him, he follows those signs; if not, he goes backward following the same path where he came from, until the external entrance/exit door is found. Visitors collected in the courtyard, will converge towards the intermediate emergency exit then, finally, they will reach the exit door. A snapshot of a possible situation created by such a dynamics after an alarm event is shown, just to give an example, in Fig.3, where the existing configuration of three emergency exits, without emergency signs, is reported. Agents are represented with colored squares with a size slightly smaller than one patch, and different colors correspond to different initial accessing groups. Since each agent, at each time step, moves from the center of a patch exactly to the center of another patch, a little empty space separates adjacent visitors (this is a necessary precaution to avoid superposition of agents and to prevent them crossing through walls).

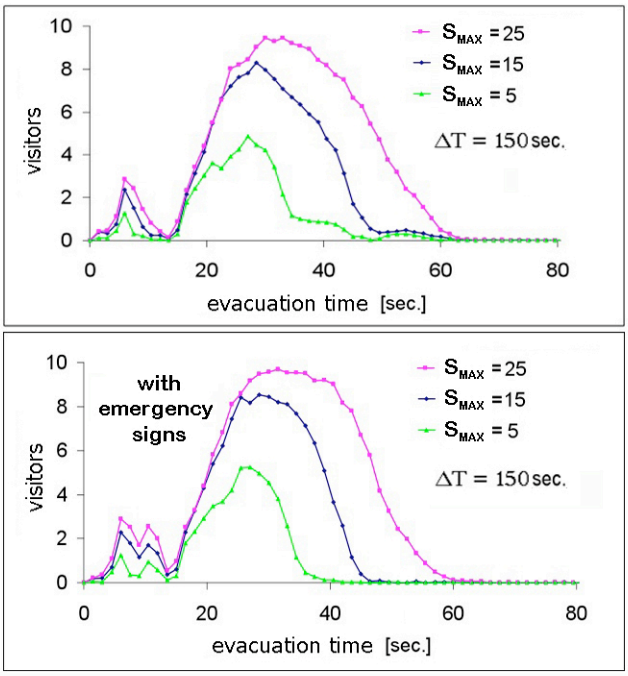

Figure 5. Alarm dynamic simulation. Distribution of the evacuation time for different group sizes in the existing configuration of three internal emergency exits, in absence of emergency signs (upper panel) and in presence of emergency signs (lower panel).

Before presenting the results about the evacuation dynamics after an alarm situation, let us show in Fig.4 an histogram where the density of visitors (number of visitors per artwork) is reported for each room and for three different maximum sizes S_{MAX} of the accessing groups, after a sufficiently long transient of normal fruition dynamics. The inter-arrival time, i.e. the interval between the entrance of one group and the next one, is ΔT=150 sec. The values reported in the figure are averaged over 100 different simulation runs, performed with the same parameters. It seems that the density in the various rooms is affected by the size of the room and by the number of exposed artworks (rooms 2 and 4, like the entrance room 10, are more than three times larger than the others). This is a quite unexpected result, which identifies big spaces as critical elements when emergency procedures have to be managed (in fact two of the three internal emergency exits are located in rooms 2 and 4). This result does not depend on the group size and we verified that it does not change by modifying the inter-arrival time. Besides, it is worthwhile to

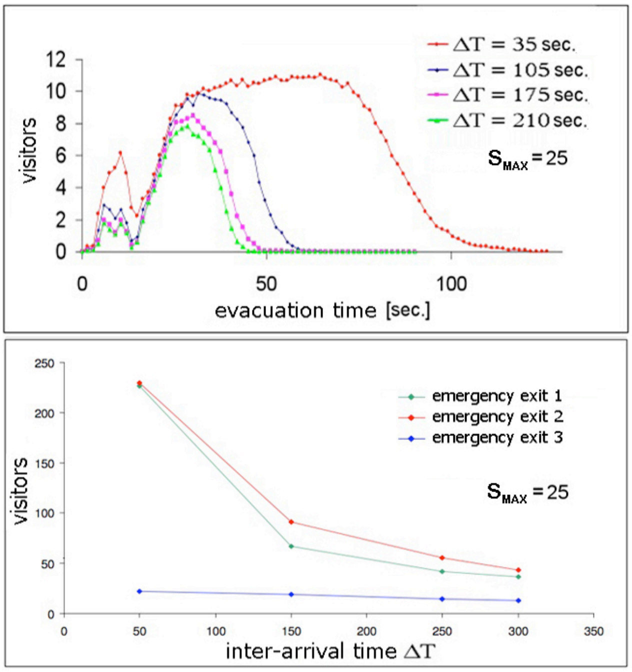

Figure 6. Alarm dynamics simulation. (Upper panel) Distribution of the evacuation time for increasing inter-arrival times in the existing configuration of three internal emergency exits; (Lower panel) Number of visitors escaping from the three internal emergency exits as function of the inter-arrival time. In the simulations of both the panels the group size is 25 and emergency signs are present.

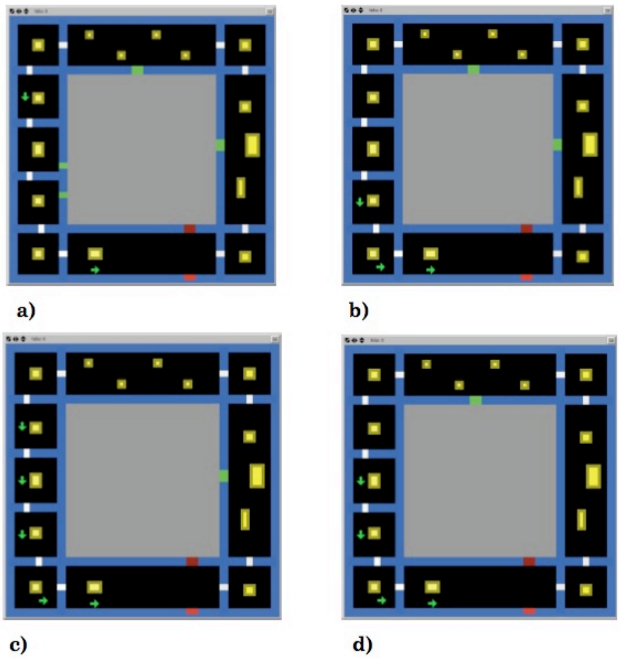
a) b) c) d)

Figure 7. Alternative configurations of internal emergency exits. a) Four emergency exits; b) Two emergency exits (1 and 2); c) Only emergency exit 1; d) Only emergency exit 2.

stress how density of visitors decreases in the last rooms, while getting closer to the end of the visit.

B. Alarm dynamics: simulation results and conclusions

Now imagine that, in a situation like the one just presented, an emergency alarm randomly goes off. First of all, let us consider how evacuation time (i.e. the time each visitor takes to reach the last external exit door from the moment when the alarm starts) depends on the group size, in the existing configuration of three internal emergency exits (see Fig.3) and no emergency signs. In these and in all the next simulations averages over 100 different runs were always performed. In the upper panel of Fig.5 the distribution of the evacuation time is reported for three different group sizes (S_{MAX}= 5, 15, 25) and for an inter-arrival time of ΔT=150 sec. In other words, the vertical axis measures how many visitors reach the exit within the time indicated in the horizontal axis. First we observe that the distribution is bimodal (i.e. with two peaks); this is due to the fact that the small number of visitors closer to the entrance are able to rapidly evacuate the museum, coming back and going out from there in a few seconds, while visitors far from the entrance need a longer time to evacuate, both for a longer distance to run (which includes also the internal courtyard path) and for the crowding around the emergency exits. Distribution stays quite similar varying the group size; the only difference, as expected, is a shift towards higher evacuation time when group size increases. The lower panel of Fig.5 shows that the introduction of emergency signs (the green arrows in rooms 6, 7 and 10 of Fig.2) slightly reduces the average values of evacuation times. In this case the distributions have a third small peak around 10 seconds, due to the contribution of visitors situated in the last rooms which, instead of coming back to the entrance, follow the emergency signs then reducing their evacuation time. In the upper panel of Fig.6 we show, in the same emergency configuration than before, how the evacuation time distribution is affected by the inter-arrival time, fixing the group size S_{MAX}=25 and in presence of emergency signs. Again, the distribution is bimodal but the centroid visibly shifts towards an higher evacuation time when the inter-arrival time decreases, i.e. when the number of visitors present inside the museum when the alarm goes off increases. In particular, for ΔT=35 sec, the distribution becomes broader and broader due to a bottleneck effect in the more crowded rooms, usually (see Fig.4) rooms 2 and 4. In these rooms people crowd in front of the internal emergency exits to reach the courtyard, so that for a while the evacuation flux, and consequently the evacuation time, stays quite constant. The results of another simulation, presented in the lower panel of Fig.6, confirm such a scenario, showing that the emergency exit 1 and 2, situated respectively in rooms 2 and 4, are the most used ones for every choice of the inter-arrival time.

Let us explore now what happens if we take into account emergency configurations alternative to the existing one, being the opportunity to compare to each other different possible evacuation plans the most useful application of the agent-based simulations. We considered four hypothetical alternative configurations of the internal emergency exits, as shown in Fig.7: configuration (a), where a further exit located in room 7 (already existing, but normally closed) has been made available for evacuation; configuration (b), where exit 3, the less used, has been closed and only the other two exits 1 and 2 were used; configurations (c) and (d), where only one exit has been continued to use, respectively exit 1 and exit 2. For each

TABLE I. COMPARISON OF EMERGENCY CONFIGURATIONS

	Description	S_{MAX}	ΔT	$< T_{EV} >$
	Existing configuration with three exits	5	300	77
a)	Four emergency exits	5	300	78
b)	Two emergency exits (1 and 2)	5	300	80
c)	Only emergency exit 1	5	300	87
d)	Only emergency exit 2	5	300	72

configuration we changed opportunely the emergency signs (indicated, as usual, with green arrows in the panels of Fig.7), but we kept fixed the group size and the inter-arrival time. We made many simulations for several values of these parameters, calculating for each of them the average value (centroid) of the respective evacuation time distributions (averaged also over 100 different runs and indicated with $< T_{EV} >$). In Table I we report only the results obtained with $S_{MAX}= 5$ and $\Delta T=300$, being the latter representative of all the others. Comparing the resulting values of $< T_{EV} >$ with the corresponding value calculated for the existing configuration with three emergency exits and with emergency signs (reported in the first row of the Table), we notice that only the configuration (d) gives a better result: this is a quite surprising finding, since it means that keeping open only exit 2 instead of three or four emergency exits overlooking the courtyard it is possible to get a smaller average evacuation time, then a better global performance of the corresponding evacuation plan. In Fig.8 we can further appreciate this conclusion by directly comparing the complete evacuation time distributions of the existing configuration (red line) with that one of the alternative (d) with only exit 2 (green line): it is clear that in the latter case a greater number of visitors are able to leave the museum more quickly than in the former one, and such a shift on the left of the distribution keeps smaller also its average value. This unexpected and unpredictable result confirms the usefulness and the effectiveness of the agent-based simulations in the design and analysis of complex social systems, fully supporting more traditional strategies already available to the engineers. Moreover, it demonstrates the power of the NetLogo platform in reproducing pedestrian motion in closed environments, turning out to be a reliable tool for exploring emergent features of crowd behavior and for testing the level of service and the safety of a public area.

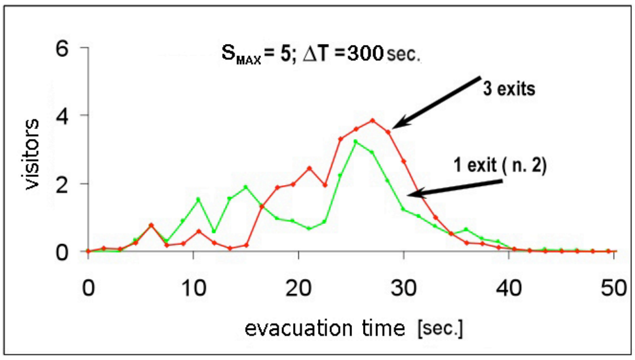

Figure 8. Comparison between the evacuation time distributions of the existing configuration with three emergency exits and the optimal alternative configuration, where only emergency exit 2 is used.

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