

*News, Expectations and Trends in
Statistical Physics, **NEXT-SigmaPhi**
3rd International Conference
13-18 Agosto 2005 – Kolymbari (Crete)*



Changing Opinions in a Changing World

A new perspective in Sociophysics

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Outline of the talk

Sociophysics and Opinion Dynamics

The HK Compromise Model

The 2-D HK Compromise Model

The Kuramoto model

**The Opinion Changing Rate model:
numerical results**

Sociophysics and Opinion Dynamics

Following the basic theorem of interdisciplinary research that states “Physicists not only know everything; they know everything better”, physicist have long tried to apply their skill to fields outside of physics, with varying degrees of success.

Biophysics, Bioinformatics and Econophysics have been progressively in fashion in the last years.

Sociophysics and Opinion Dynamics have been around for at least three decades, with or without that name.

Sociophysics and Opinion Dynamics

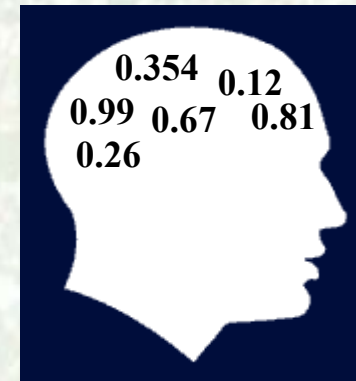
The majority of opinion dynamics models developed in the last years (Sznajd, Deffuant, Hegselmann and Krause, Galam, Stauffer etc.) try to answer to the following question:

“Is it possible to put in agreement agents having different opinions?”

In all above-mentioned models opinions are modeled as **numbers** (integer or real).

Sociophysics and Opinion Dynamics

Of course the **reduction of humans opinions to simple numbers** is a great simplification, and cognitive scientist might dislike it.



But such a dispute sounds like the **reduction of Earth to a point mass** in the Kepler Laws. Clearly, the Earth is not point-like, but for the purposes of describing celestial motions **this approximation was good** and led to the development of theoretical mechanism by Newton and others.

Sociophysics and Opinion Dynamics

Furthermore, in analogy with statistical mechanics laws, if the behaviour of a person is essentially **unpredictable**, the global organization of many mutually interacting subjects presents **general patterns** which go beyond specific individual attributes and may emerge in several different contexts.



Sociophysics and Opinion Dynamics

Therefore one can suppose that, in a sociophysics context, quantities like **averages and statistical distributions** may characterize not just specific situations but large classes of system.

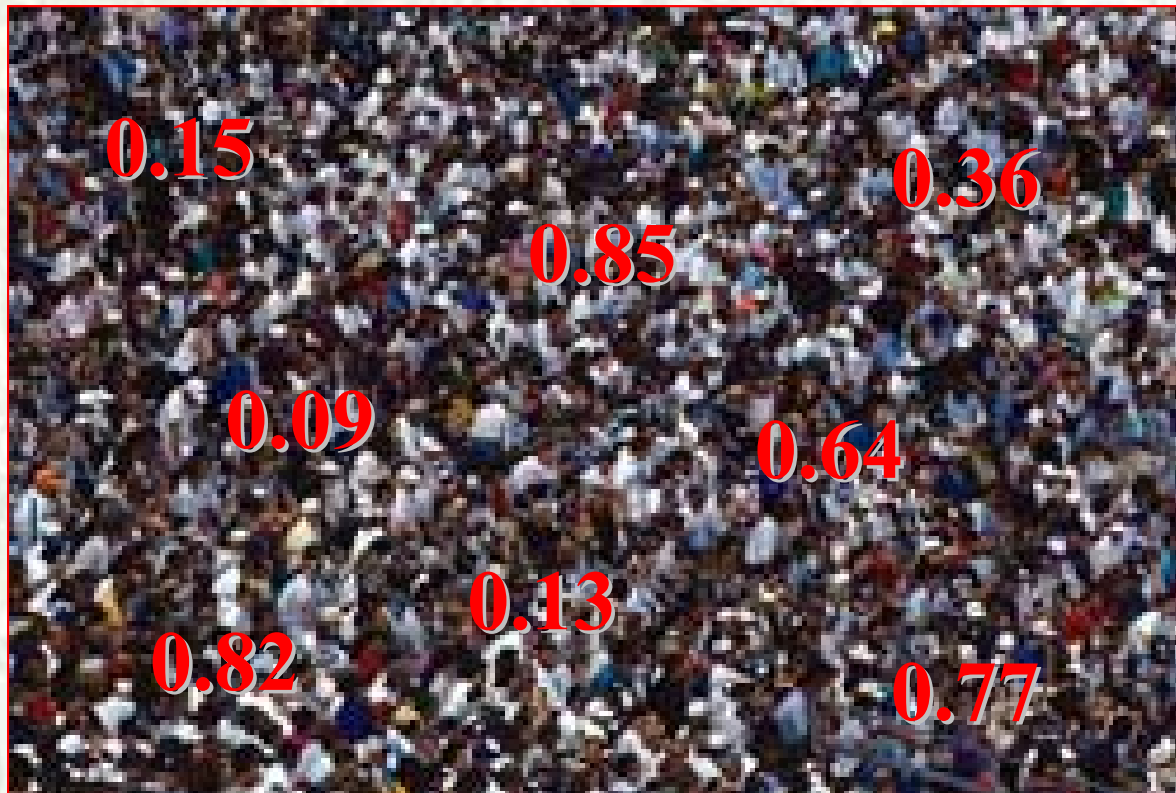


Sociophysics and Opinion Dynamics

Usually, in opinion dynamics models, one starts by assigning **randomly** a number (i.e. an opinion) to every agent of a given population...



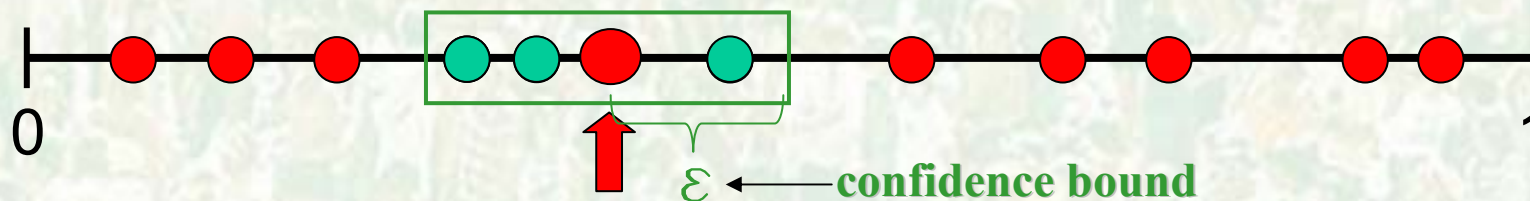
...then the dynamics starts to act, and the agents **rearrange their opinion** variables due to mutual discussion.



Sociophysics and Opinion Dynamics: the HK model

The **Hegselmann-Krause (HK)** model* is based on the presence of a parameter ε , called “**confidence bound**”, which expresses the ‘range of compatibility’ of the agents’ opinions

The **1-D opinion space** is represented by the points of a $[0,1]$ line, where the agents’ opinions are randomly distributed:



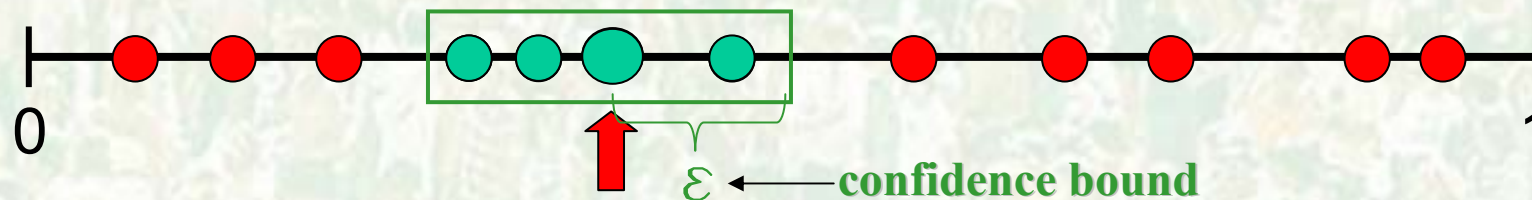
At each step, one chooses at random **one of the opinions**, corresponding to a given agent, and checks how many opinions are compatible with it, i.e. are **inside the confidence bound**...

*R. Hegselmann and U. Krause, Journal of Artificial Societies and Social Simulation 5, issue 3, paper 2 (jasss.soc.surrey.ac.uk) (2002);

Sociophysics and Opinion Dynamics: the HK model

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...then the new opinion of the selected agent becomes equal to the **average opinion** of its compatible neighbours in the opinion space.

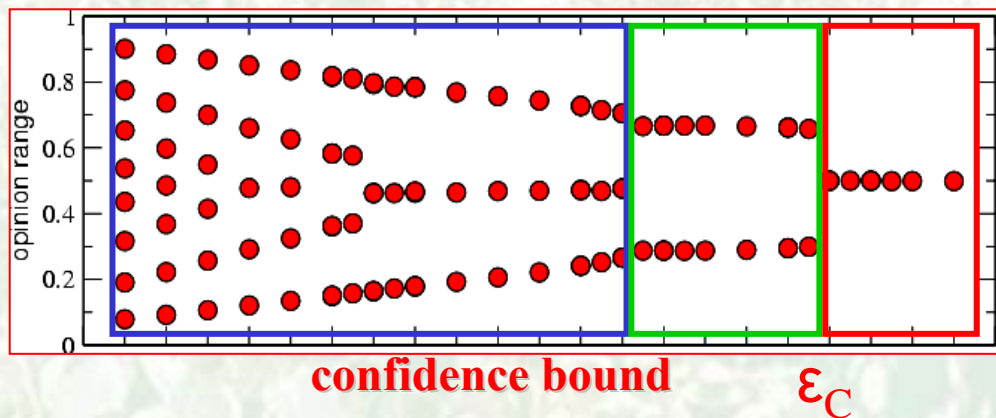
*R. Hegselmann and U. Krause, Journal of Artificial Societies and Social Simulation 5, issue 3, paper 2 (jasss.soc.surrey.ac.uk) (2002);

Sociophysics and Opinion Dynamics: the HK model

The HK dynamics clearly tends to clusterize opinions. The type of **final stationary clusters configuration** reached by the system depends on the value of the confidence bound .

This final configuration may represent:

- **Fragmentation**, where several opinion clusters survive
- **Polarization**, with two main clusters of opinions ("parties")
- **Consensus**, with all agents sharing the same opinion



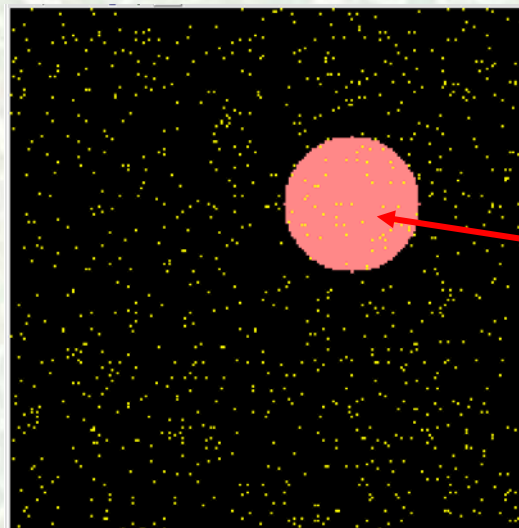
By means of Monte Carlo simulations it has been found that, for fully interacting agents, **consensus** is reached only above a critical threshold of the **confidence bound**: $\epsilon_C \sim 0.2$

*R. Hegselmann and U. Krause, Journal of Artificial Societies and Social Simulation 5, issue 3, paper 2 (jasss.soc.surrey.ac.uk) (2002)

Sociophysics and Opinion Dynamics: the 2D HK model

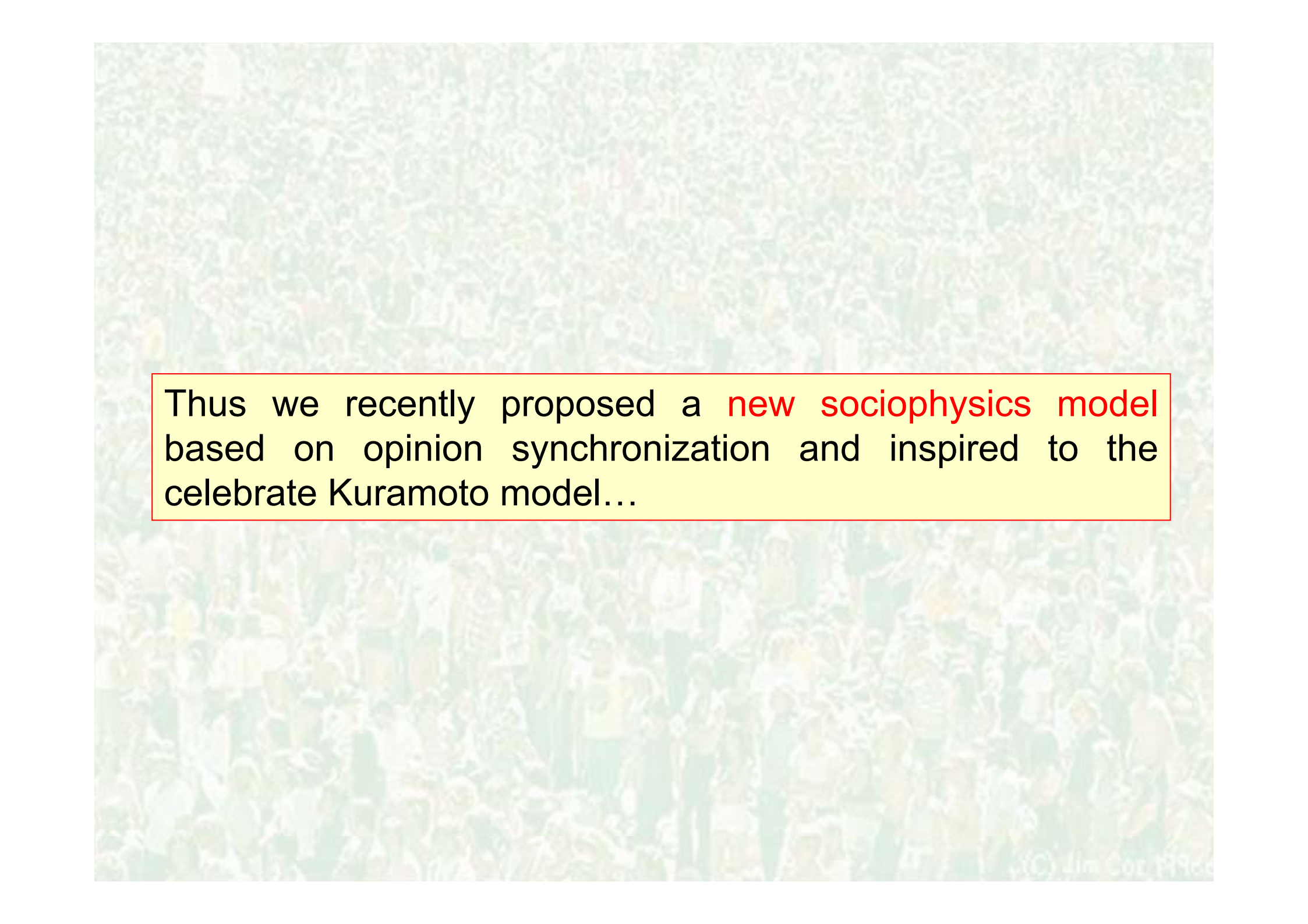
Very recently, by integrating a rate equation for a continuum distribution of opinions, we found* that, in the **HK model** with **2D squared opinion space** and circular confidence range, **consensus** is reached above the critical threshold $\varepsilon_c \sim 0.23$

Squared opinion space



circular
confidence
range

*Fortunato, Latora, Pluchino, Rapisarda, “*Vector Opinion Dynamics in a bounded confidence consensus model*” (2005) - Int.Journ.of Mod.Phys.C, in press



Thus we recently proposed a **new sociophysics model** based on opinion synchronization and inspired to the celebrate Kuramoto model...

The Kuramoto model*

The Kuramoto model is the simplest models for **synchronization** available on the market and consists of N **coupled phase oscillators** with natural frequencies ω_i and coupling parameter K :

$$\frac{d\vartheta_i(t)}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\vartheta_j - \vartheta_i), \quad i = 1, \dots, N$$

$\vartheta_i(t) \in [0, 2\pi)$

Labels in the diagram:

- ω_i : natural frequencies
- K : coupling strength
- $\vartheta_j - \vartheta_i$: phases of oscillators



The coherence of the system is measured by the mean field **order parameter** r ($0 \leq r(t) \leq 1$):

$$r e^{i\psi} = \frac{1}{N} \sum_{j=1}^N e^{i\vartheta_j}$$

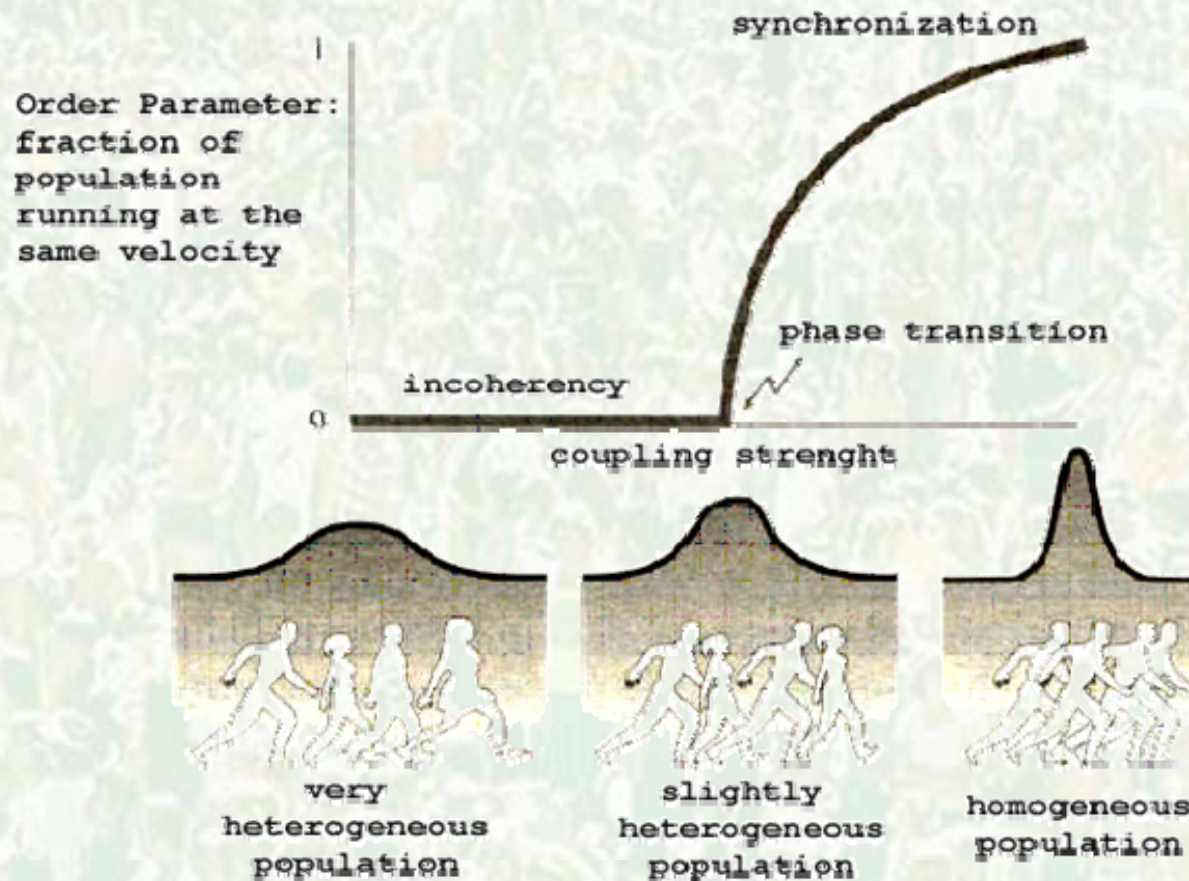
Through r the rate equations can be rewritten as:

$$\frac{d\vartheta_i(t)}{dt} = \omega_i + Kr \sin(\psi - \vartheta_i), \quad i = 1, \dots, N$$

*proposed by Y.Kuramoto in 1975

The Kuramoto model

As Kuramoto showed analitically in a beautiful analysis, one observes **synchronization** above a certain **critical value** of the control parameter K_c ...



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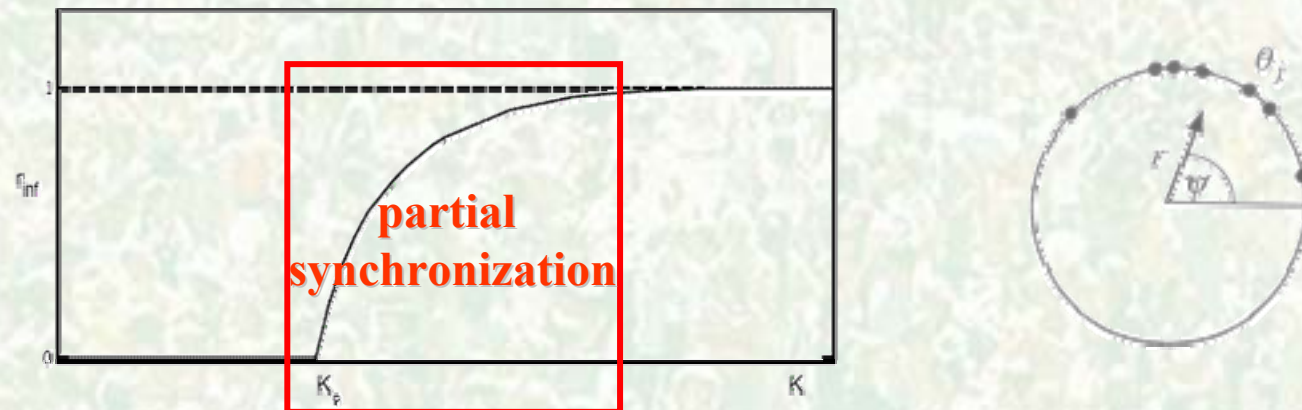


Fig. 1. Asymptotic order parameter r_{∞} as a function of the coupling in the Kuramoto model

$$K \rightarrow 0 \quad \vartheta_i(t) \approx \omega_i t + \vartheta_i(0) \quad r \rightarrow 0$$

Incoherent phase

$$K \rightarrow \infty \quad \vartheta_i(t) \approx \psi(t) \quad r \rightarrow 1$$

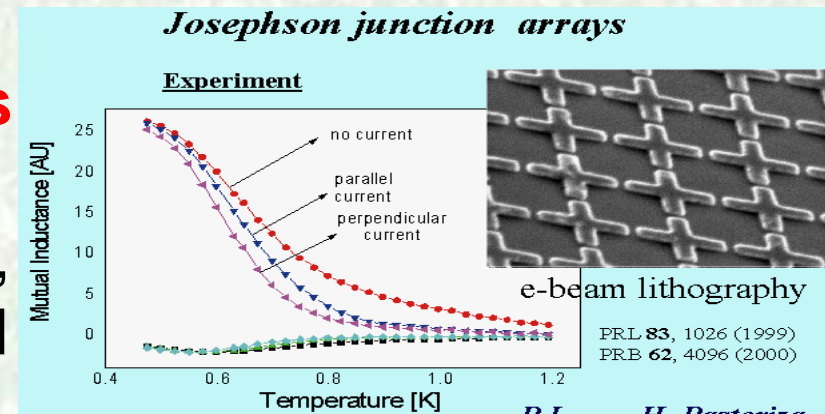
Global synchronization

The Kuramoto model

Many applications to:

Physical or Chemical systems

(Josephson junction arrays,
Landau damping in plasmas,
chemical oscillators, coupled
laser arrays, ...)



Biological systems

(fireflies, pacemaker cells in
the heart and in the brain,
chorusing crickets, ...)

The Kuramoto model

Summing up, the Kuramoto model is **simple enough** to be mathematically tractable, yet **sufficiently complex** to be not-trivial...

In the following we will try to apply the Kuramoto model to

SOCIAL SYSTEMS

in the context of **Sociophysics** and, in particular, of **Opinion Dynamics** models...

Sociophysics and Opinion Dynamics

Inspired by the **Kuramoto model**, we propose a new consensus model based on the opinion synchronization of many agents affected by an individual **different inclination** to change...

Sociophysics and Opinion Dynamics

Actually, the world changes and we change with it...

...but everyone in a different way:

-There are **conservative** people, that tend to maintain their opinion or their style of life against everything and everyone;

-There are **more flexible** people that change idea quite easily and usually follow any current fashion and trend;

-Finally there are those who **run faster** than the rest of the world anticipating the others with new ideas and insights (**progressist** or **innovative** people).

Sociophysics and Opinion Dynamics

Thus the true **question** to answer should not be:

“Is it possible to put in agreement agents having a different natural inclination to change opinion?”

...but should become:

“Is it possible to put in agreement agents having different opinions?”

The Opinion Changing Rate model*

In order to do this, we **modified** the Kuramoto model considering the following rate equations describing N interacting agents

$$\frac{dx_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(x_j - x_i) e^{-\alpha|x_j - x_i|}, \quad i = 1, \dots, N$$

Annotations for the equation:

- coupling strenght** (K)
- natural op.changing rates** (ω_i)
- opinions** ($x_j - x_i$)

$$x_i(t) \in]-\infty, +\infty[$$

$$\omega_i \in [0, 1] \text{ time independent!}$$

- the $x_i(t)$ are the agent's opinions
- the ω_i are the so-called **natural opinion changing rate**, i.e. the natural (fixed) tendency of the *i*-th agent to change its opinion, uniformly distributed. This allow us to simulate **conservative** ($\omega_i \sim 0$) and **innovative** people ($\omega_i \sim 1$).

*Pluchino, Latora, Rapisarda, Int.Journ.of Mod.Phys.C (2005)

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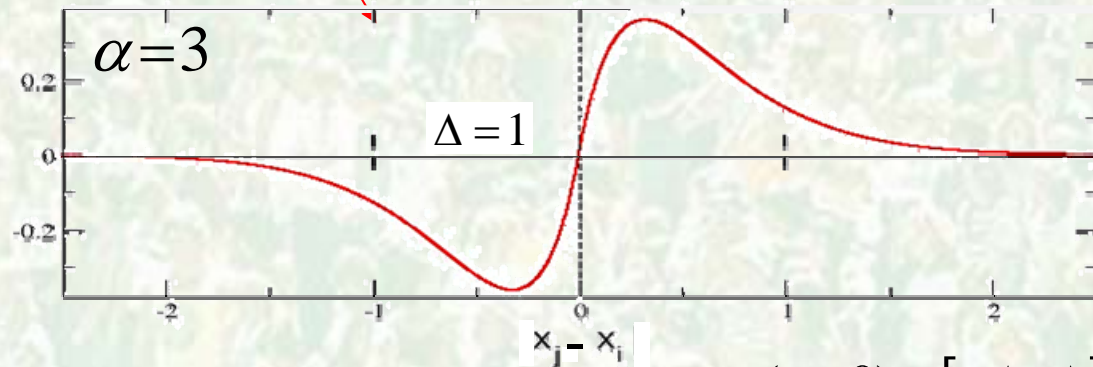
coupling strenght
opinions

natural op.changing rates
opinions

$$x_i(t) \in]-\infty, +\infty[$$

$$\omega_i \in [0, 1] \text{ time independent!}$$

The **interaction potential** decreases for distant opinions:

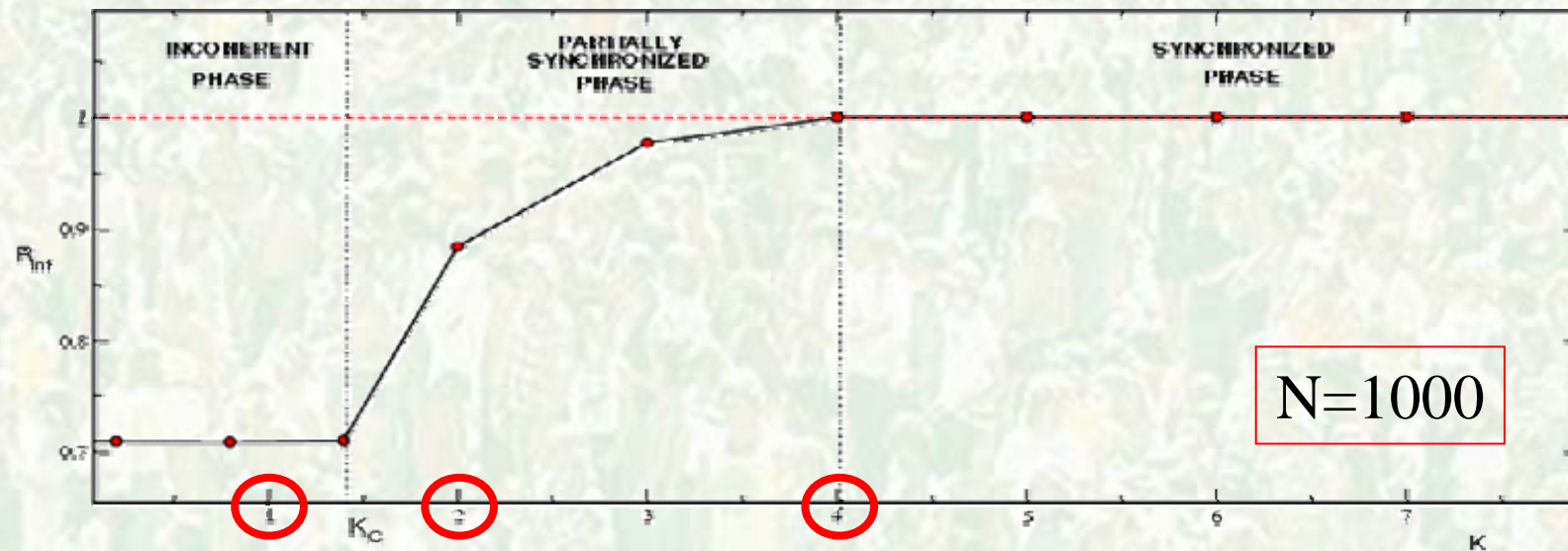


$$x_i(t=0) \in [-\Delta, \Delta]$$

*Pluchino, Latora, Rapisarda, Int.Journ.of Mod.Phys.C (2005)

The Opinion Changing Rate model

Defining a coherence **order parameter** R by means of the standard deviation of the opinion changing rate, we observe a **Kuramoto-like phase transition** :

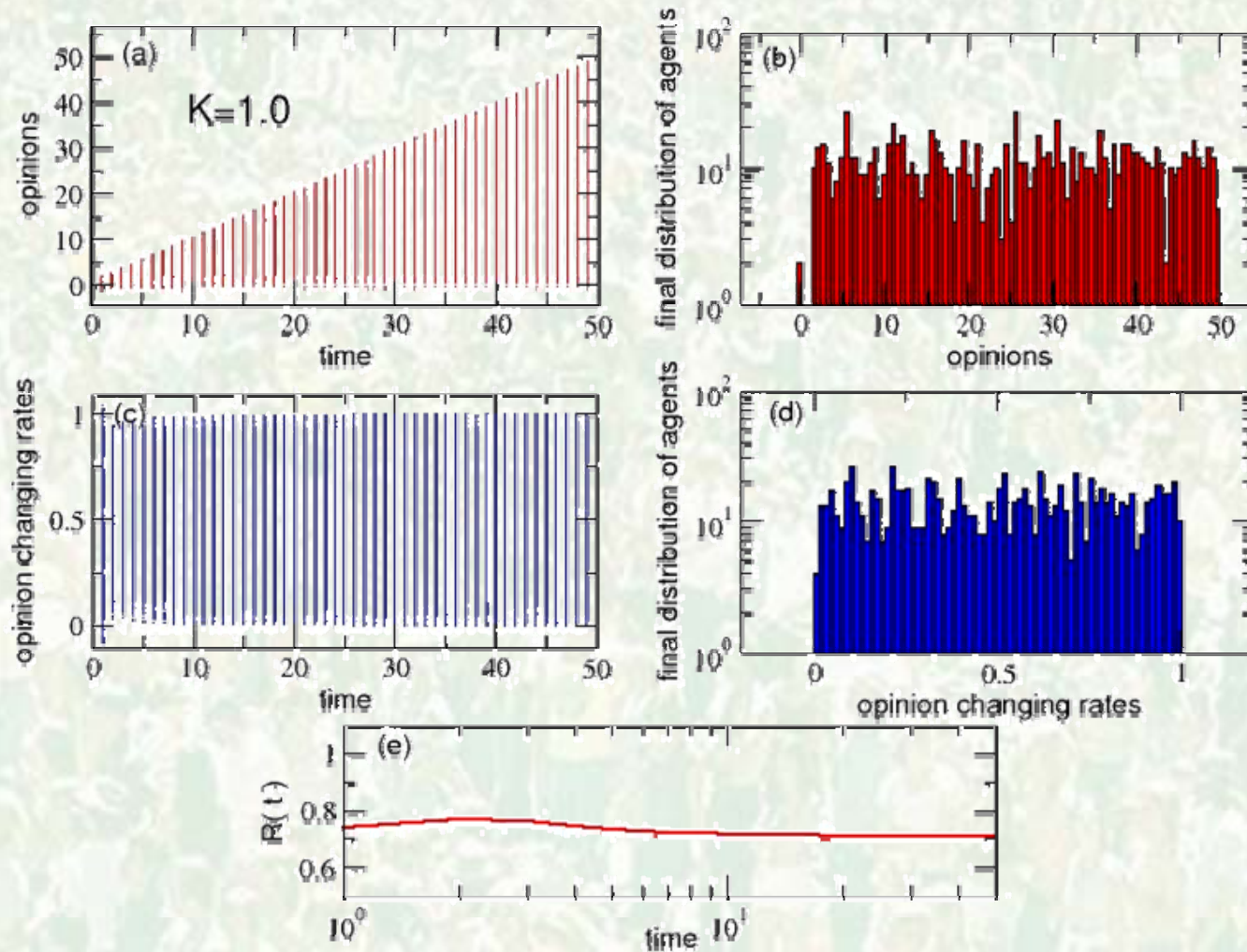


Phase transition for the asymptotic order parameter R_{inf} at $K_C \sim 1.4$

OCR: numerical results (1)

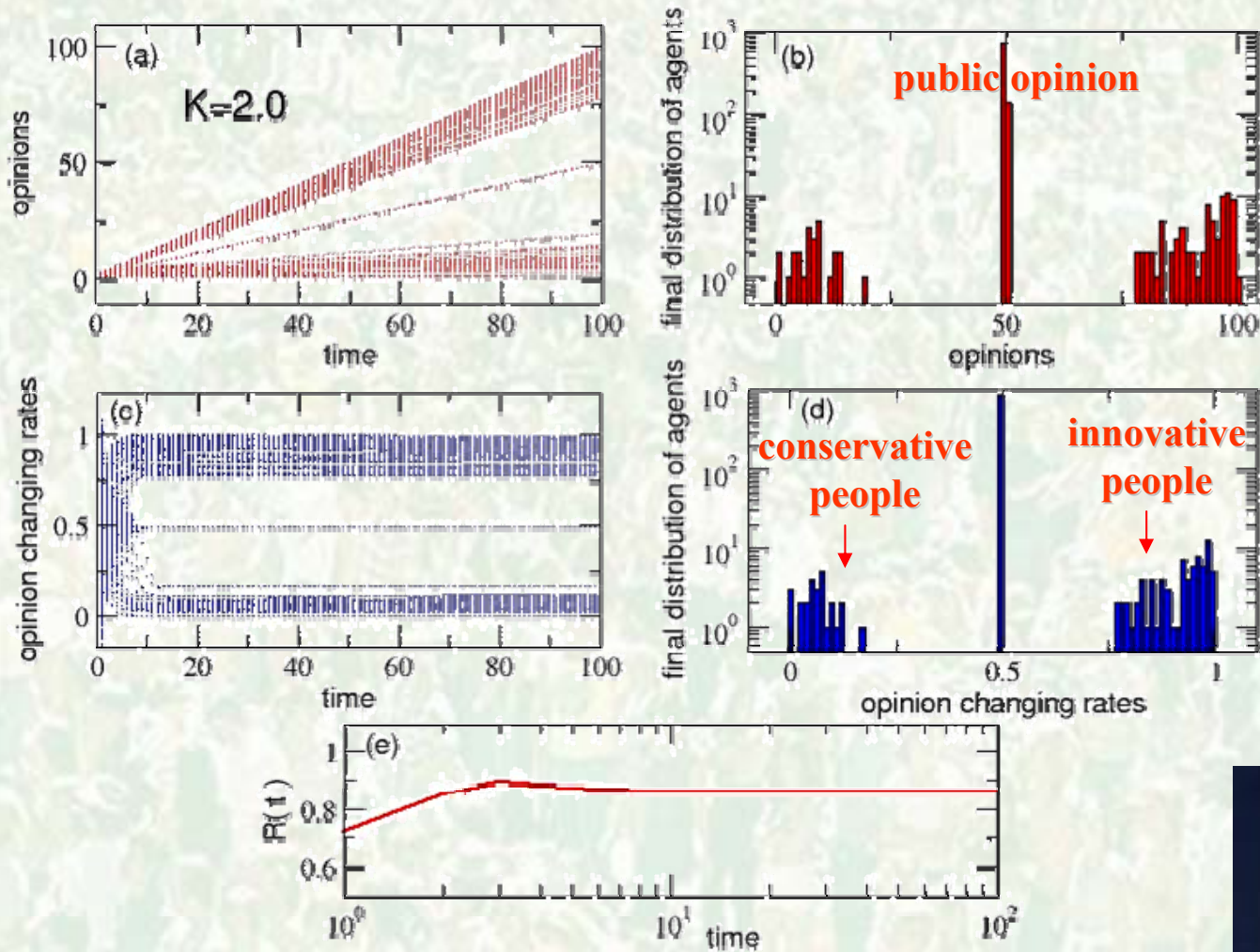
K=1 (incoherent phase) : anarchy?

$N=1000$



OCR: numerical results (2)

K=2 (partially synch. phase) : bipolarism?

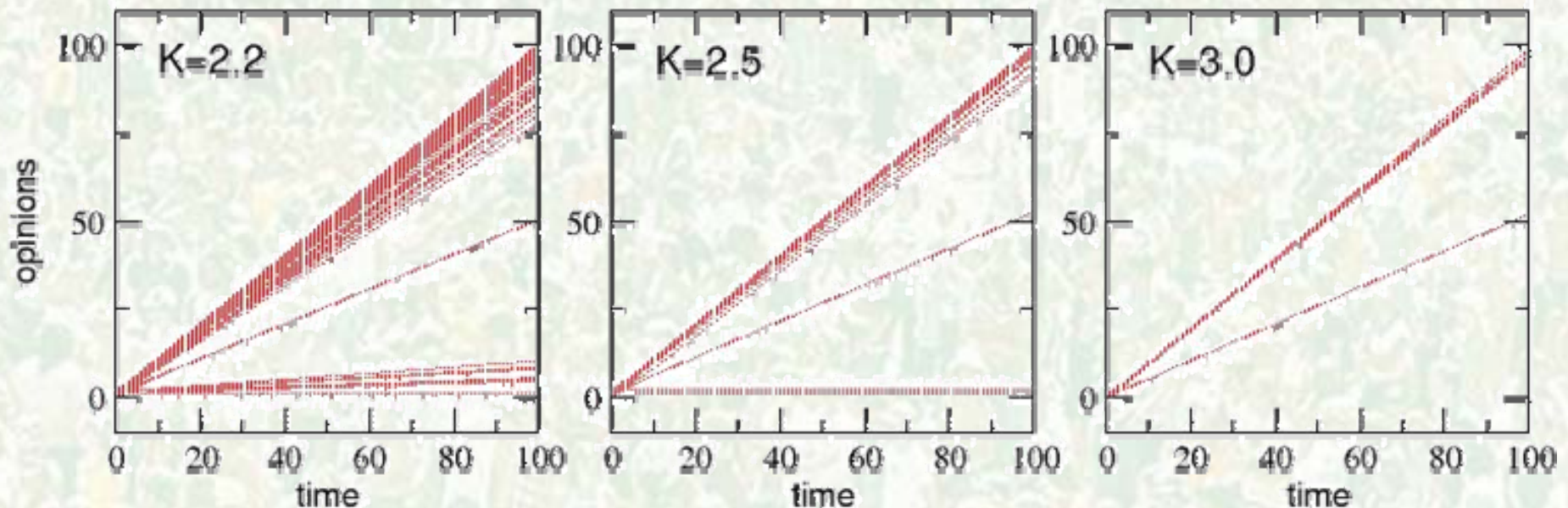


$N=1000$



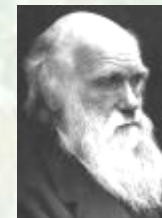
OCR: numerical results (3)

K= 2.2 - 2.5 - 3.0 (partially synch. phase)



Increasing K in the partially synchronized phase the innovative group survives longer than the conservative one... **Why?**

“It is not the strongest that survives, nor the most intelligent; it is the one that is the most adaptable to change”
C.Darwin

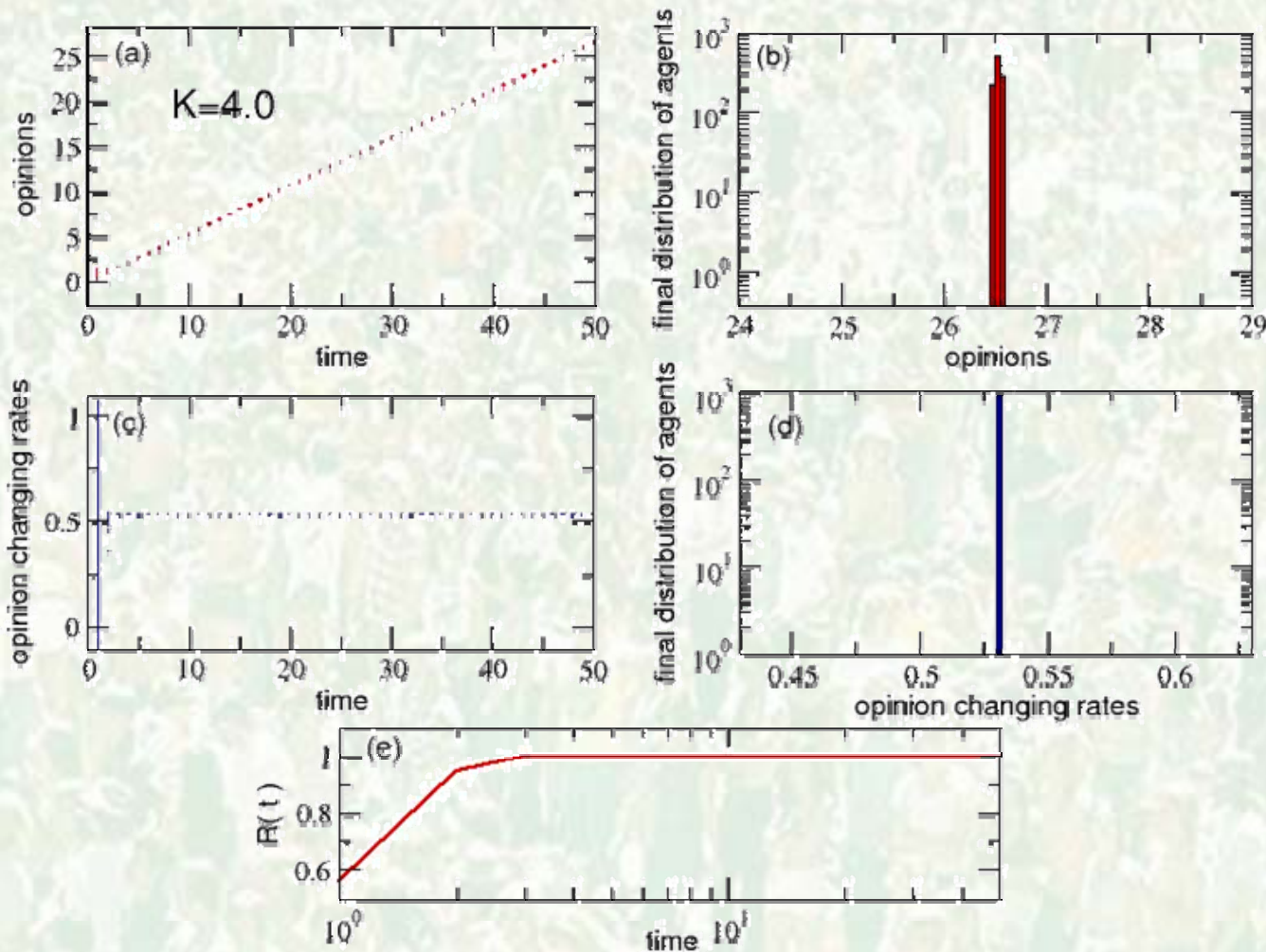


Jim Cor 1960

OCR: numerical results (4)

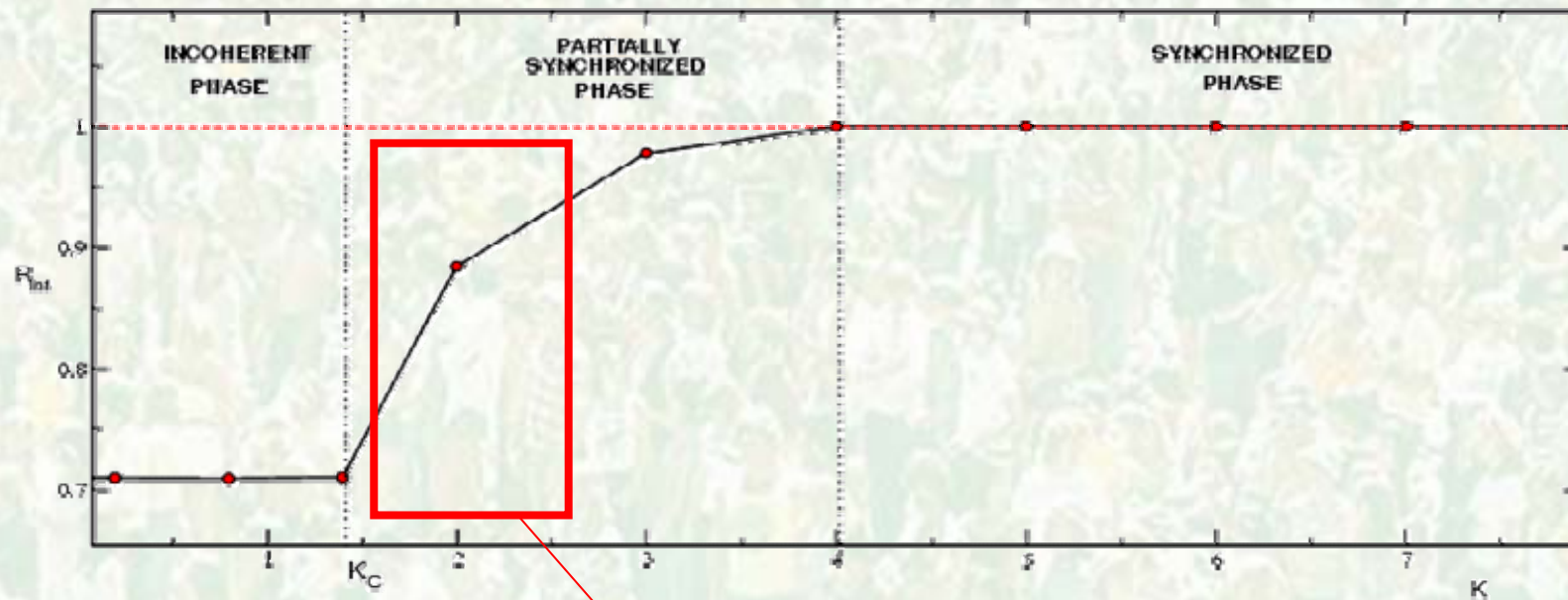
K=4 (synchronized phase) : dictatorship?

$N=1000$



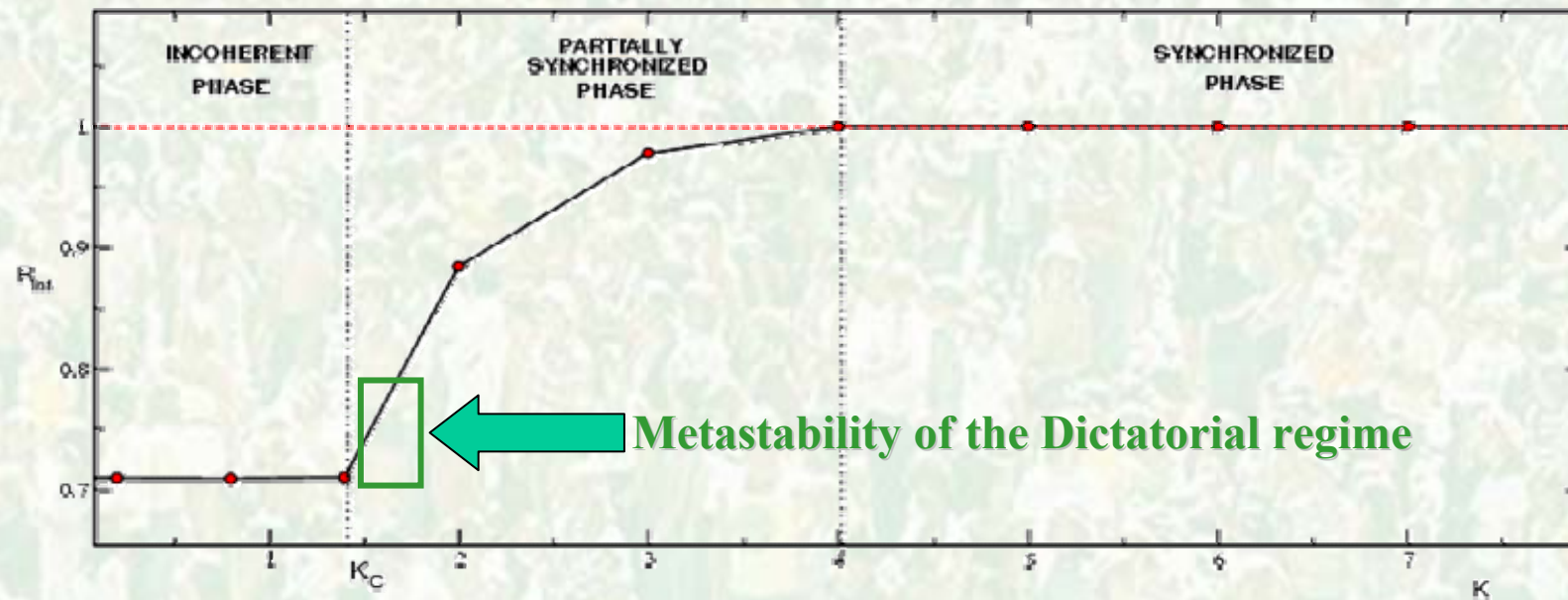
OCR: numerical results (5)

Thus, in order to ensure an equilibrium between conservative and innovative groups (**democracy**), a changing society needs a **coupling K** strictly included in a narrow window ($1.5 < K < 2.5$)



Democracy window

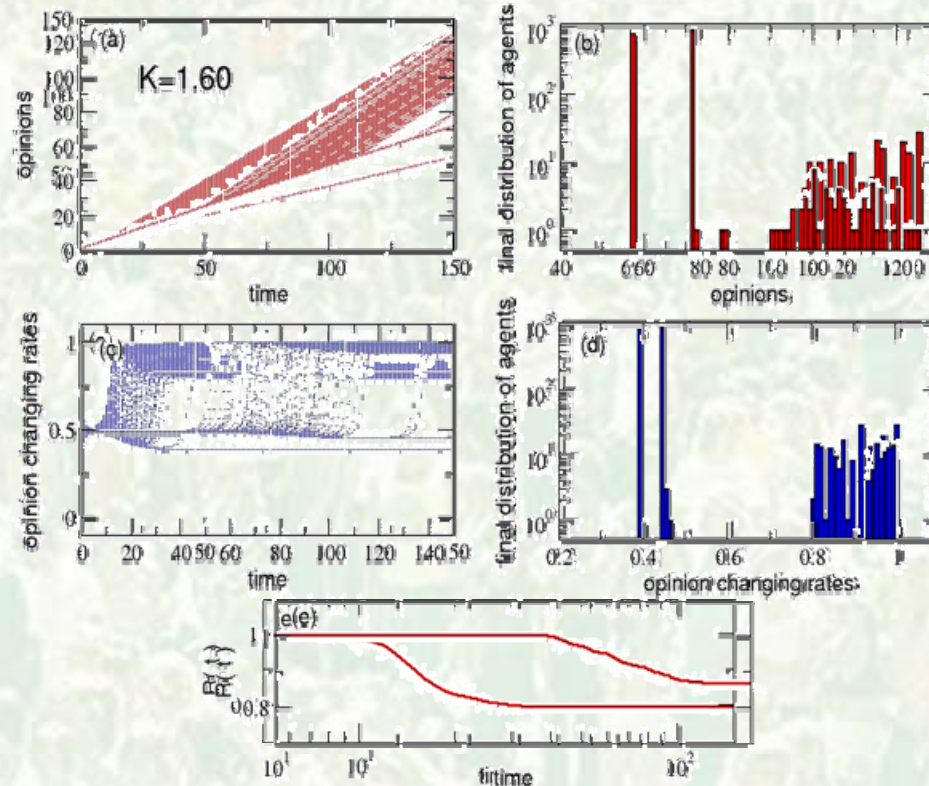
OCR: numerical results (5)



OCR: numerical results (6)

Metastability of the dictatorship regime

If one starts all the agents with the same opinion (dictatorship) at the beginning of the partially synchronized phase, one observes a metastability regime that becomes stable approaching the value $K=1.62$

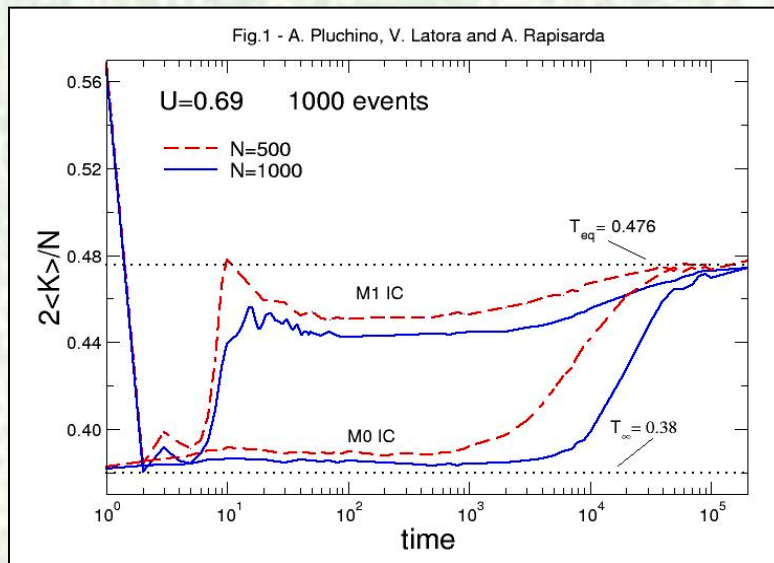


$N=1000$

OCR: numerical results (7)

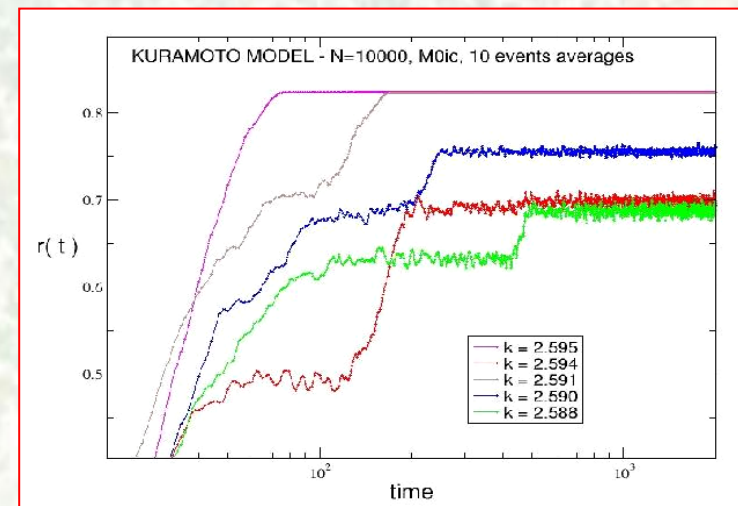
Metastability near the phase transition seems to be ubiquitous in many models:

Hamiltonian Mean Field Model



Pluchino, Latora, Rapisarda, *Physica D* 193 (2004) 315 ; *Physica A* 338 (2004) 60

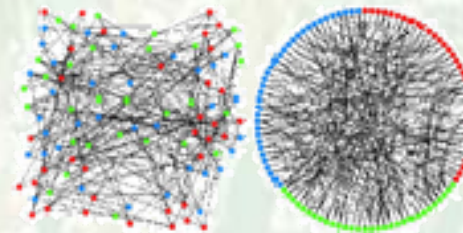
Kuramoto Model



Pluchino, Latora, Rapisarda, 'Metastability hindering synchronization in HMF and Kuramoto models' in preparation

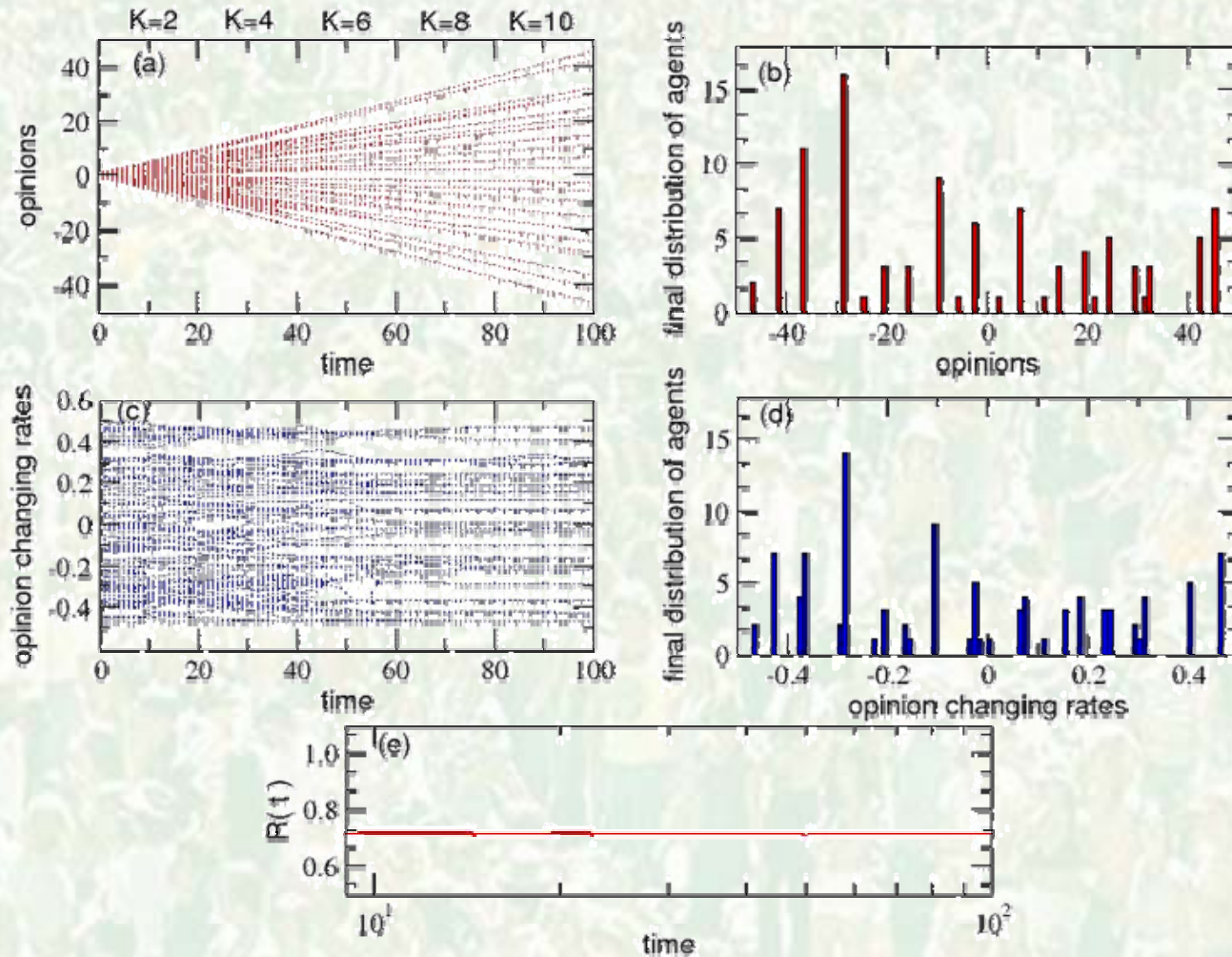
K-Satisfiability Model

Mezard, Parisi, Zecchina, 'Analytic and Algorithmic Solution of Random Satisfiability Problems' - *Science* 279 (2002) p.842



OCR: numerical results (8)

Increasing coupling: from anarchy to democracy?

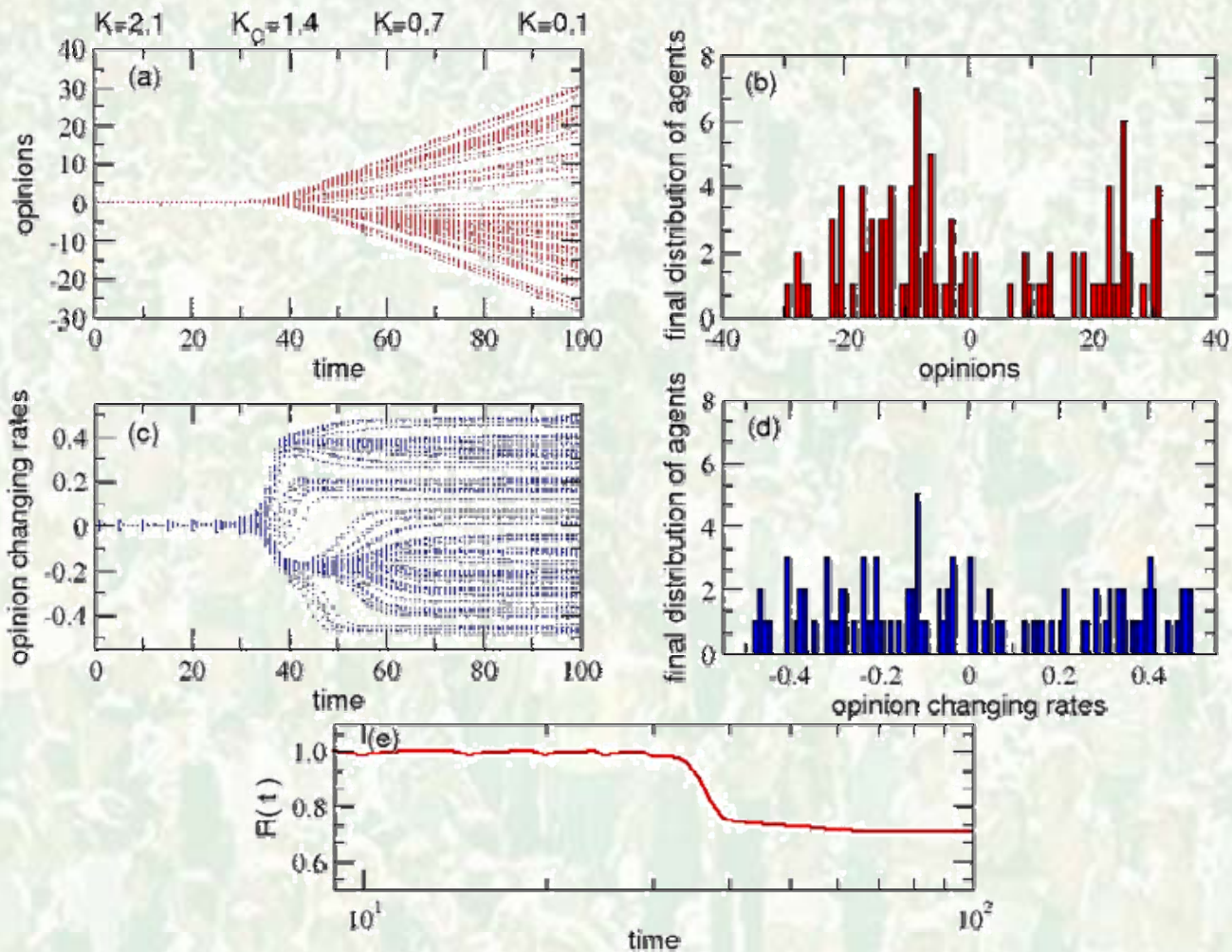


$N=100$

Political parties
or
opinion clusters
formation

OCR: numerical results (9)

Decreasing coupling: from order to anarchy?



$N=100$

Fall of a
dictatorship
or
dissolution of
an empire

In conclusion:

- The OCR model extends the standard sociophysics equilibrium context and focus on the dynamical aspects of opinion formation
- In spite of its simplicity, it seems to capture many general features of the opinion formation process

Future developments of OCR model:

- social agents on a complex network topology
- introduction of an external field (mass-media pressure)
- addition of disorder (frustration) in the coupling
- parallelization of the integration algorithm



<http://www.ct.infn.it/~cactus>