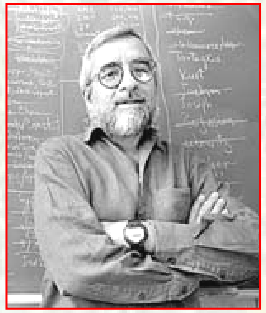
Nonlinearity, Nonequilibrium and Complexity: questions and perspectives in statistical physics

Tepoztlán, Mexico, Nov.27th to Dec. 2nd 2005,
in celebration of Alberto Robledo's sixtieth birthday (HAPPY BIRTHDAY ALBERTO!!!)



The Role of Synchronization in Sociophysics

From Opinion Dynamics to Community Structures Identification

A.Pluchino

in collaboration with A. Rapisarda, V. Latora, S. Fortunato* and S. Boccaletti**

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**Indiana University* ***Florence University*

Outline of the talk

Sociophysics and Opinion Dynamics:

The 2D-Hegselmann Krause Compromise Model

Synchronization and Opinion Dynamics:

From Kuramoto to the Opinion Changing Rate model

Finding Community Structures in Complex Networks:

Tuning Synchronization in Weighted Networks

Sociophysics and Opinion Dynamics:

The 2D-Hegselmann Krause Compromise Model

Following the basic theorem of interdisciplinary research that states “Physicists not only know everything; they know everything better”, physicists (the only ones that believe in this theorem!:-) have long tried to apply their skill to fields outside of physics, with varying degrees of success.

Biophysics, Bioinformatics and Econophysics have been progressively in fashion in the last years.

Actually, Sociophysics and Opinion Dynamics have been around for at least three decades, with or without that name…

The majority of opinion dynamics models developed in the last years (Sznajd, Deffuant, Hegselmann and Krause, Galam, Stauffer etc.) try to answer to the following question:

“Is it possible to put in agreement agents having different opinions?”

In all above-mentioned models opinions are modelized as numbers (integer or real).

Of course the reduction of humans opinions to simple numbers is a great semplification, and cognitive scientist might dislike it.

But such a dispute sounds like the reduction of Earth to a point mass in the Keplero Laws. Clearly, the Earth is not point-like, but for the purposes of describing celestial motions this approximation was good and led to the development of theoretical mechanism by Newton and others.

Furthermore, in analogy with statistical mechanics laws, if the behaviour of a person is essentially unpredictable, the global organization of many mutually interacting subjects presents general patterns which go beyond specic individual attributes and may emerge in several different contexts.

Therefore one can suppose that, in a sociophysics context, quantities like averages and statistical distributions may characterize not just specific situations but large classes of systems…

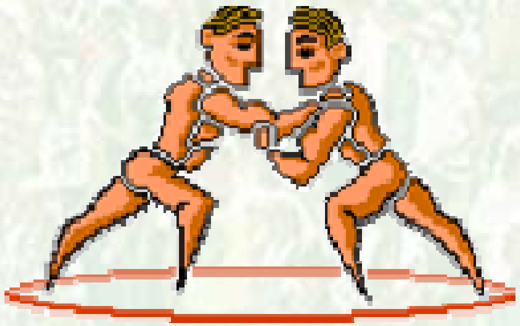
Sociophysics and Opinion Dynamics

Usually, in opinion dynamics models, one starts by assigning randomly a number (i.e. an opinion) to each agent of a given population (distributed over a network in the physical space)…

…then the dynamics starts to act, and the agents rearrange their opinion variables (in the opinion space) due to mutual discussion .

Thus the fundamental question in standard opinion dynamics is:

"Under what conditions is it possible to put in agreement agents having different opinions?"

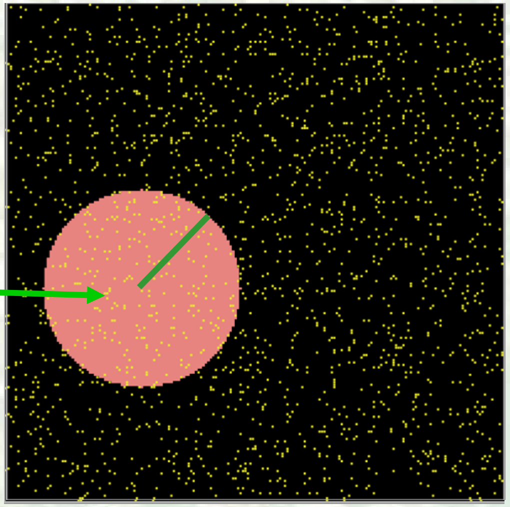


The Hegselmann and Krause model

The Hegselmann-Krause (HK) model* is based on the presence of a parameter ε, called "confidence bound", which expresses the 'range of compatibility' of the agents' opinions

In the fully coupled **2D**-HK model each opinion is a two-dimensional vector represented by a point in a [0,1]x[0,1] squared opinion space:

circular confidence range, with radius ε



At each step, one chooses at random (or sequentially) one opinion, corresponding to a given agent, and checks how many opinions are compatible with it, i.e. are inside the confidence range…

*R. Hegselmann and U. Krause, Journal of Articial Societies and Social Simulation 5, issue 3, paper 2 (jasss.soc.surrey.ac.uk) (2002);

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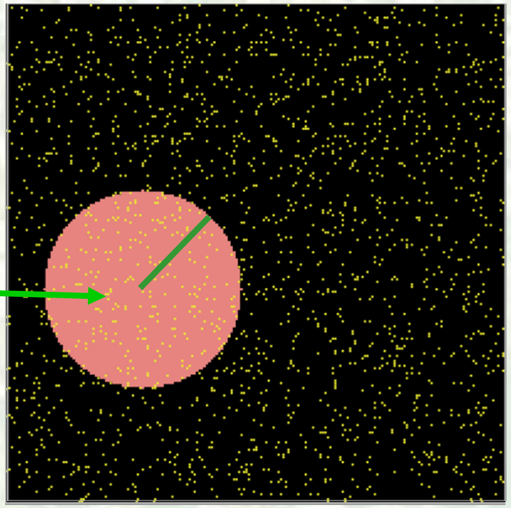
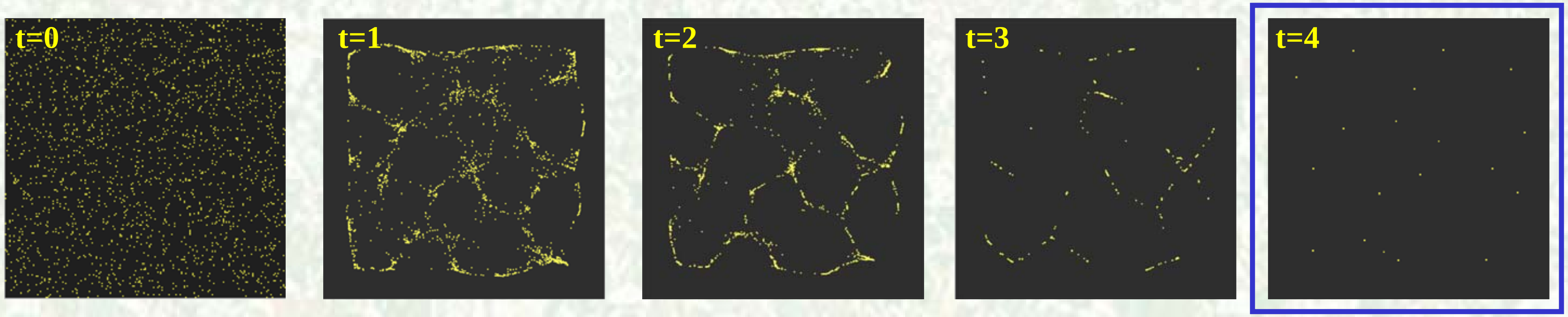


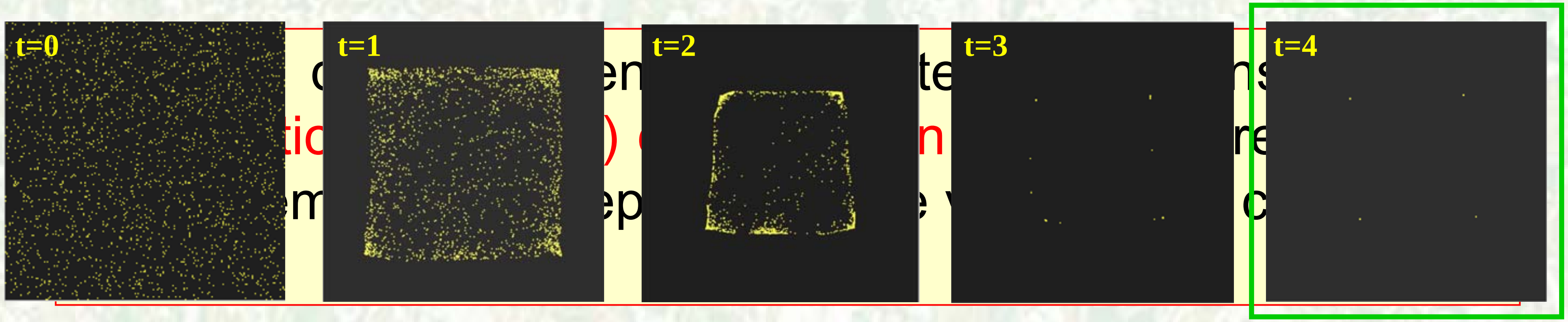
…then the new opinion of the selected agent becomes equal to the **average opinion** of its compatible neighbours.

*R. Hegselmann and U. Krause, Journal of Articial Societies and Social Simulation 5, issue 3, paper 2 (jasss.soc.surrey.ac.uk) (2002);

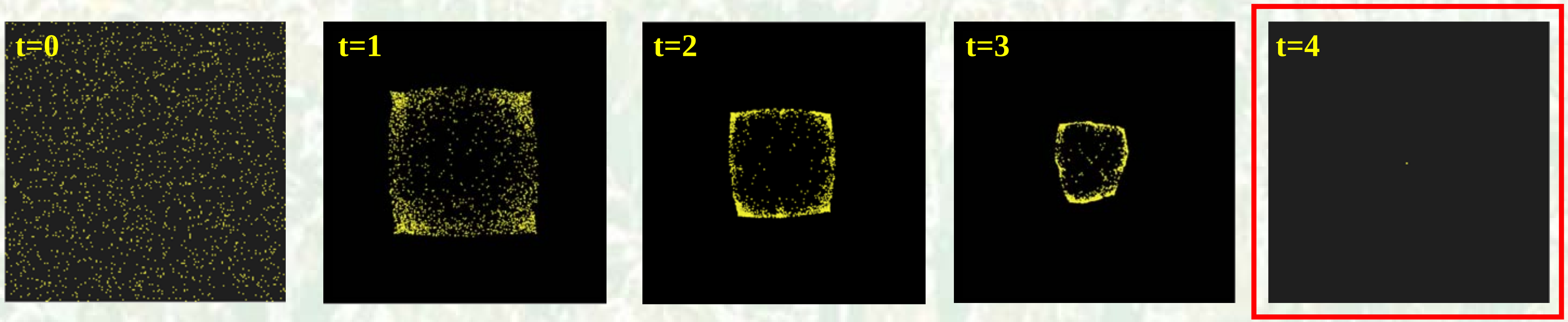
ε=0.10 : **Fragmentation**, where several opinion clusters survive



ε=0.20 : **Polarization**, with few big clusters of opinions ("parties") survive



ε=0.30 : **Consensus**, with all agents sharing the same opinion

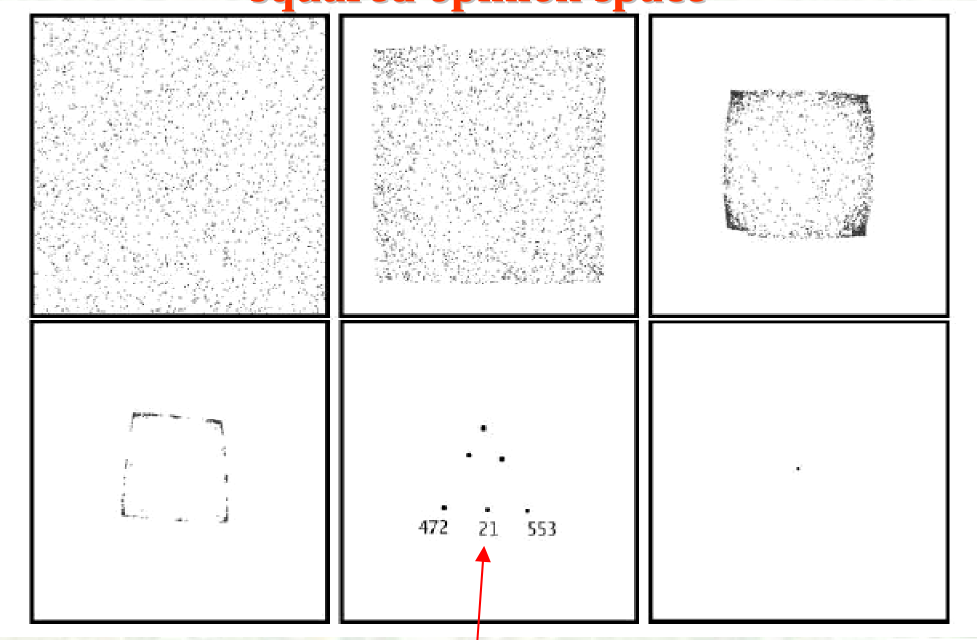


Discrete Monte Carlo (MC) simulations with N=2000 fully connected agents and simultaneous sequential update

By means of Monte Carlo simulations we found that in the 2D-HK model with squared opinion space consensus is reached above the critical threshold $\varepsilon_c \sim 0.24$, a value that tends to $\varepsilon_c \sim 0.23$ in the limit of an infinite number of agents.

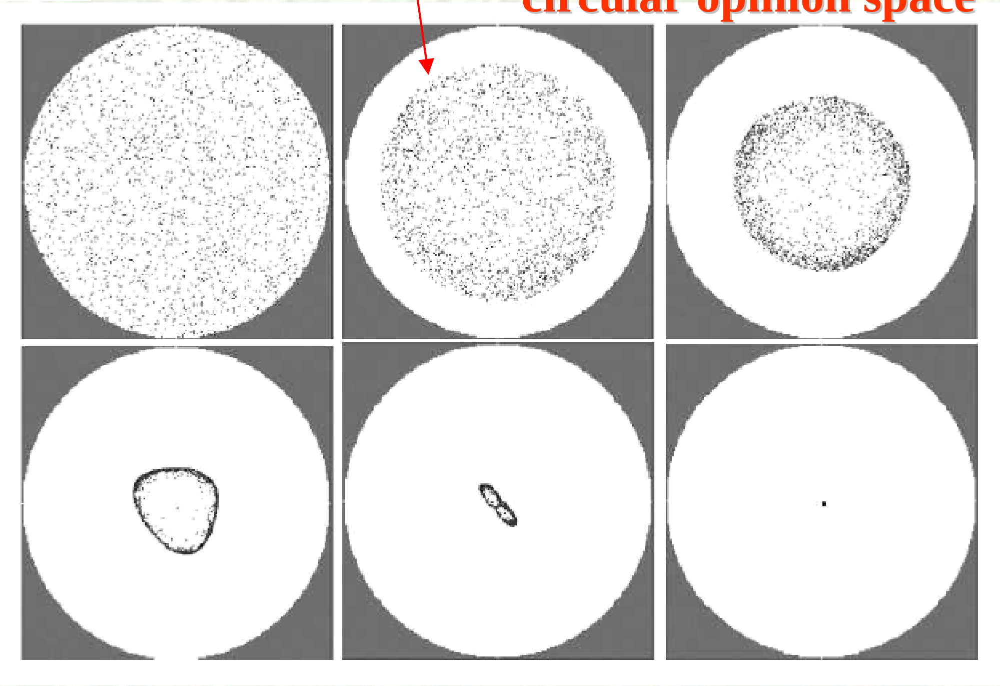
A.Pluchino, V.Latora and A.Rapisarda, Proceedings of the 3rd Int.Conf. NEXT $\Sigma\Phi$ - Kolymbari, Creta (2005)

squared opinion space



Dynamics always starts to act from the edges of the opinion space, thus the shape of the opinion space rules the simmetry of the cluster evolution…

circular opinion space



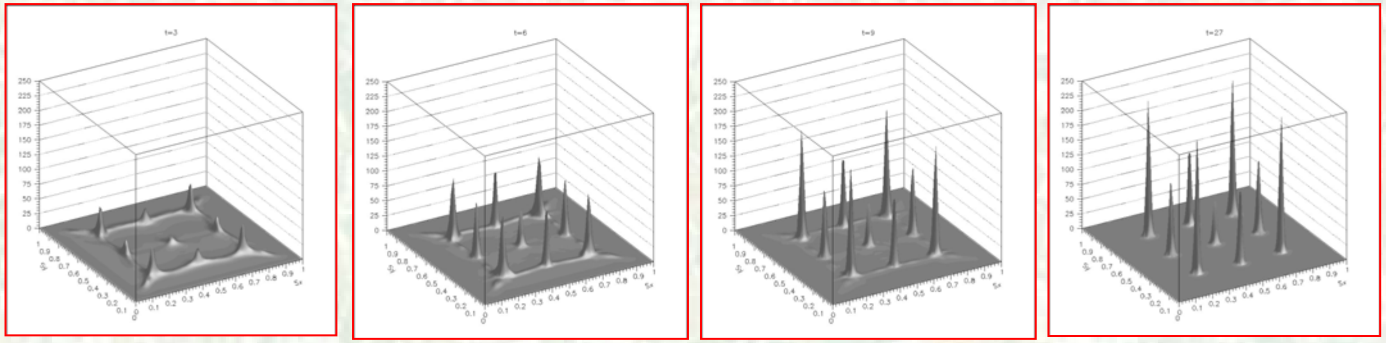
Very often, in Monte Carlo simulations, consensus is reached through the so called "connectors", little groups of people that make a bridge between otherwise not interacting social groups…

A.Pluchino, V.Latora and A.Rapisarda, Proceedings of the 3rd Int.Conf. NEXT ΣΦ - Kolymbari, Creta (2005)

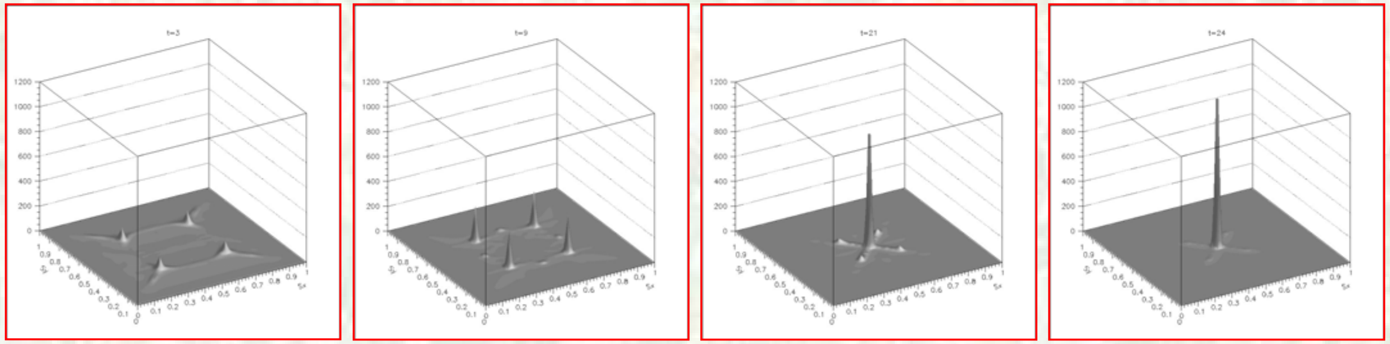
Very recently, by integrating a rate equation for a continuum distribution of 2D opinions $P(\vec{x},t)$ - that simulates an infinite number of agents - , we found* that, in the HK model with squared opinion space, consensus is reached above the critical threshold $\varepsilon_c \sim 0.23$, in agreement with MC results

$$\frac{\partial}{\partial t}P(\vec{x},t) = \int_0^1 d\vec{x}_1\, P(\vec{x}_1,t)\left[\delta\left(\vec{x} - \frac{\int_{\Omega(\vec{x}_1)} d\vec{x}_0\,\vec{x}_0\,P(\vec{x}_0,t)}{\int_{\Omega(\vec{x}_1)} d\vec{x}_0\,P(\vec{x}_0,t)}\right) - \delta(\vec{x}-\vec{x}_1)\right]$$

Below the consensus threshold



Above the consensus threshold



*Fortunato, Latora, Pluchino, Rapisarda, Int.Journ.of Mod.Phys.C, **16** (2005) 1535

Thus we recently proposed a new sociophysics model based on opinion synchronization and inspired to the celebrate Kuramoto model…

Synchronization and Opinion Dynamics:

From Kuramoto to the Opinion Changing Rate model

The Kuramoto model*

The Kuramoto model is the simplest models for synchronization available on the market and consists of N coupled phase oscillators with natural frequencies ω_i :

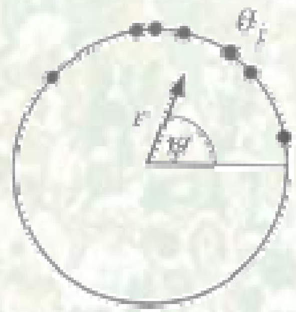
coupling strenght

$$\frac{d\vartheta_i(t)}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\vartheta_j - \vartheta_i) \;, \quad i = 1, \ldots, N$$

$\vartheta_i(t) \in [0, 2\pi)$

fixed natural frequencies

phases of oscillators



The coherence of the system is measured by the mean field order parameter r $(0 \le r(t) \le 1)$:

$$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\vartheta_j}$$

*proposed by Y.Kuramoto in 1975

The Kuramoto model

As Kuramoto showed analitically in a beautiful analysis, one observes phase synchronization above a given *critical threshold* of the control parameter K_c …

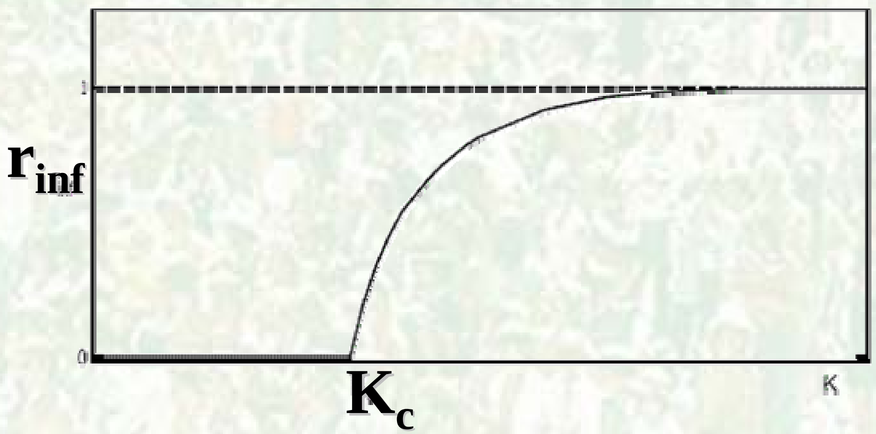
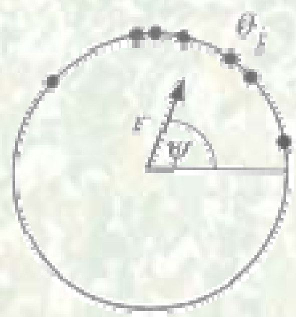


Fig. 1. Asymptotic order parameter r_∞ as a function of the coupling in the Kuramoto model



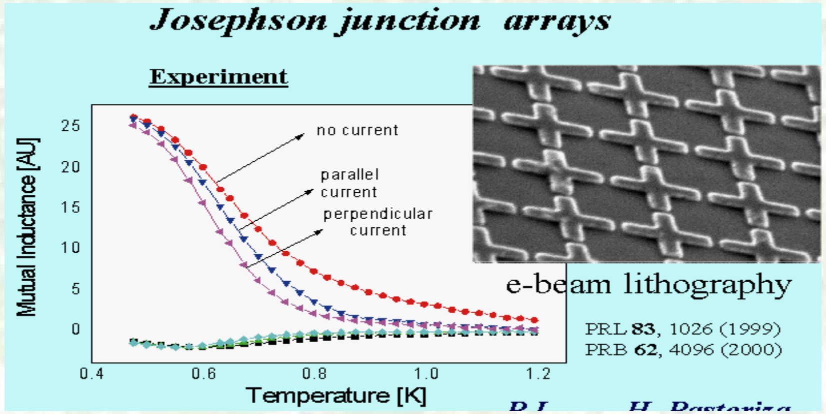
$$K \to 0 \qquad \vartheta_i(t) \approx \omega_i t + \vartheta_i(0) \qquad r \to 0$$ Incoherent phase

$$K \to \infty \qquad \vartheta_i(t) \approx \psi(t) \qquad r \to 1$$ Global synchronization

Applications of Kuramoto model

Summing up, the Kuramoto model is simple enough to be mathematically tractable, yet sufficiently complex to be not-trivial...

Physical or Chemical systems
(Josephson junction arrays, Landau damping in plasmas, chemical oscillators, coupled laser arrays, …)



Biological systems
(fireflies, pacemaker cells in the heart and in the brain, chorusing crickets, …)

Actually, the world changes and we change with it…

…but everyone in a different way:

-There are conservative people, that tend to maintain their opinion or their style of life against everything and everyone;

-There are more flexible people that change idea quite easily and usually follow any current fashion and trend;

-Finally there are those who run faster than the rest of the world anticipating the others with new ideas and insights (progressist or innovative people).

Inspired by the Kuramoto model, we proposed a new consensus model based on the opinion synchronization of many agents affected by an individual different inclination to change opinion (the analogous of the Kuramoto's natural frequencies)

Thus the true question to answer should not be:

~~**"Is it possible to put in agreement agents having a different natural inclination to change opinion?"**~~

…but should become:

"Is it possible to put in agreement agents having different opinions?"

The Opinion Changing Rate model*

In order to do this, we modified the Kuramoto model considering the following rate equations describing N interacting agents*:

coupling strenght

$$\frac{dx_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(x_j - x_i)\, e^{-\alpha|x_j - x_i|}, \quad i = 1,\dots.N$$

natural op.changing rates

opinions

$$x_i(t) \in \left]-\infty, +\infty\right[$$

$\omega_i \in [0, 1]$ time independent!

- the $x_i(t)$ are the agent's opinions
- the ω_i are the so-called *natural opinion changing rate*, i.e. the natural (fixed) tendency of the *i-th* agent to change its opinion, uniformly distributed. This allow us to simulate conservative ($\omega_i \sim 0$) and innovative people ($\omega_i \gg 0$).

*Pluchino, Latora, Rapisarda, Int.Journ.of Mod.Phys.C **16** (2005) 515

The Opinion Changing Rate model*

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$$x_i(t) \in \left]-\infty, +\infty\right[$$

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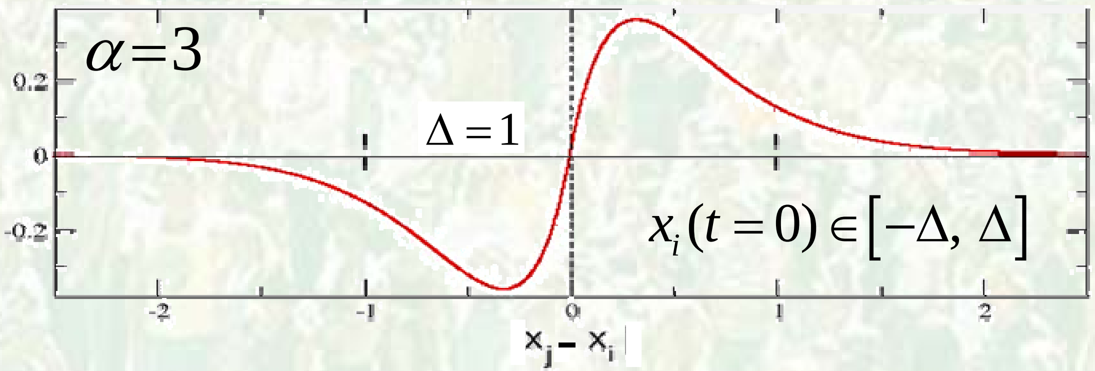
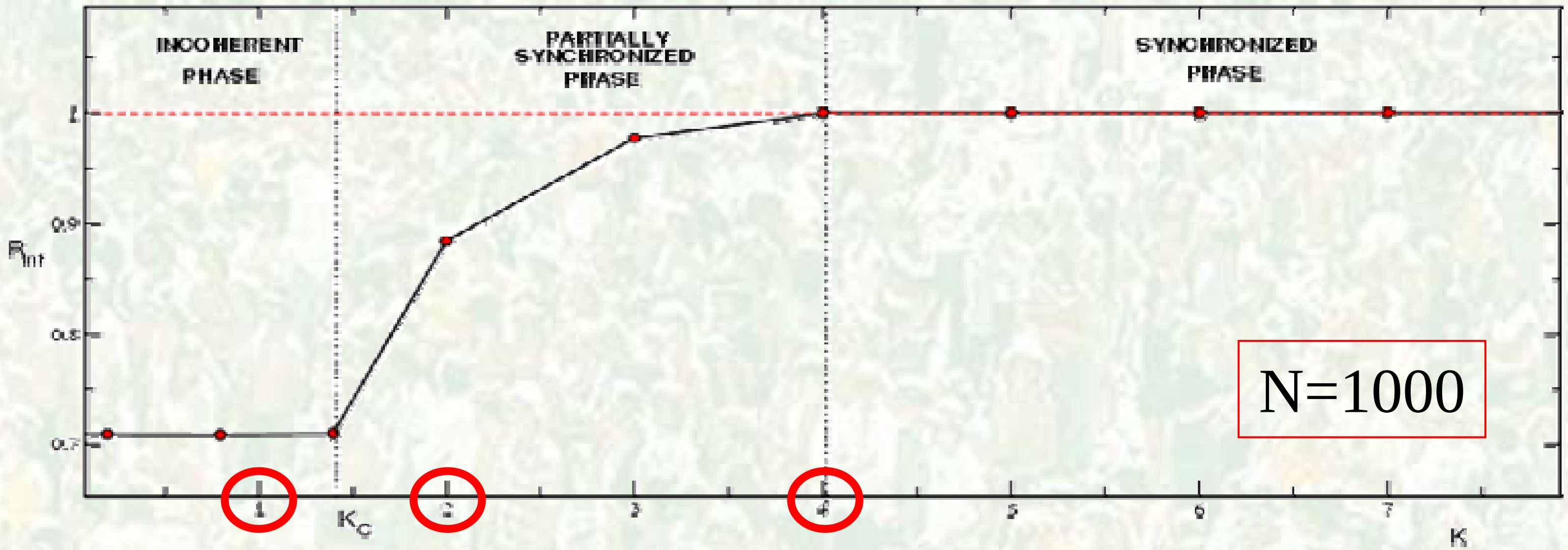
The interaction potential decreases for distant opinions:



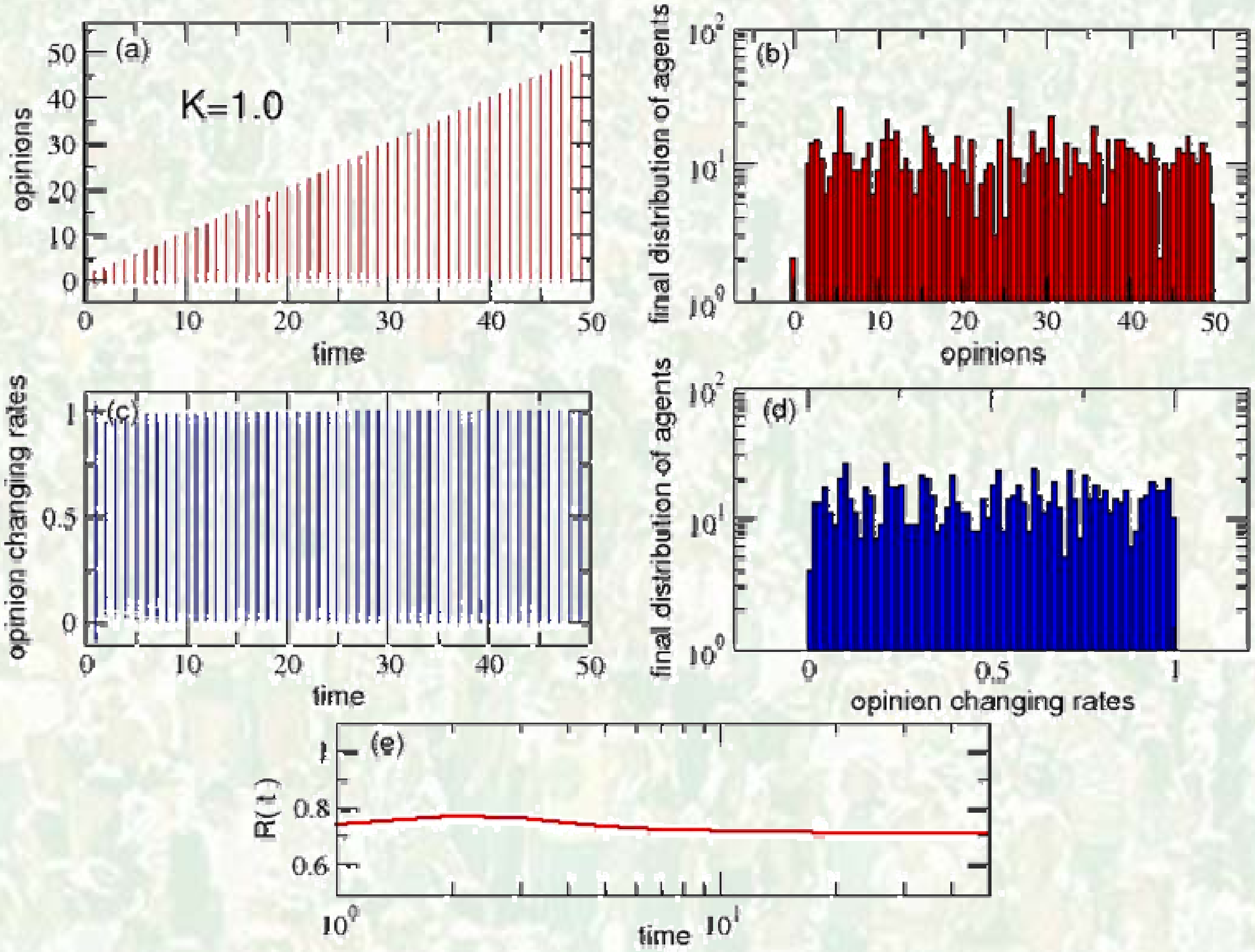
*Pluchino, Latora, Rapisarda, Int.Journ.of Mod.Phys.C **16** (2005) 515

Defining a coherence order parameter R by means of the standard deviation of the opinion changing rate (0<R<1), we observe a Kuramoto-like phase transition :



Phase transition for the asymptotic order parameter R_{inf} at K_C~1.4

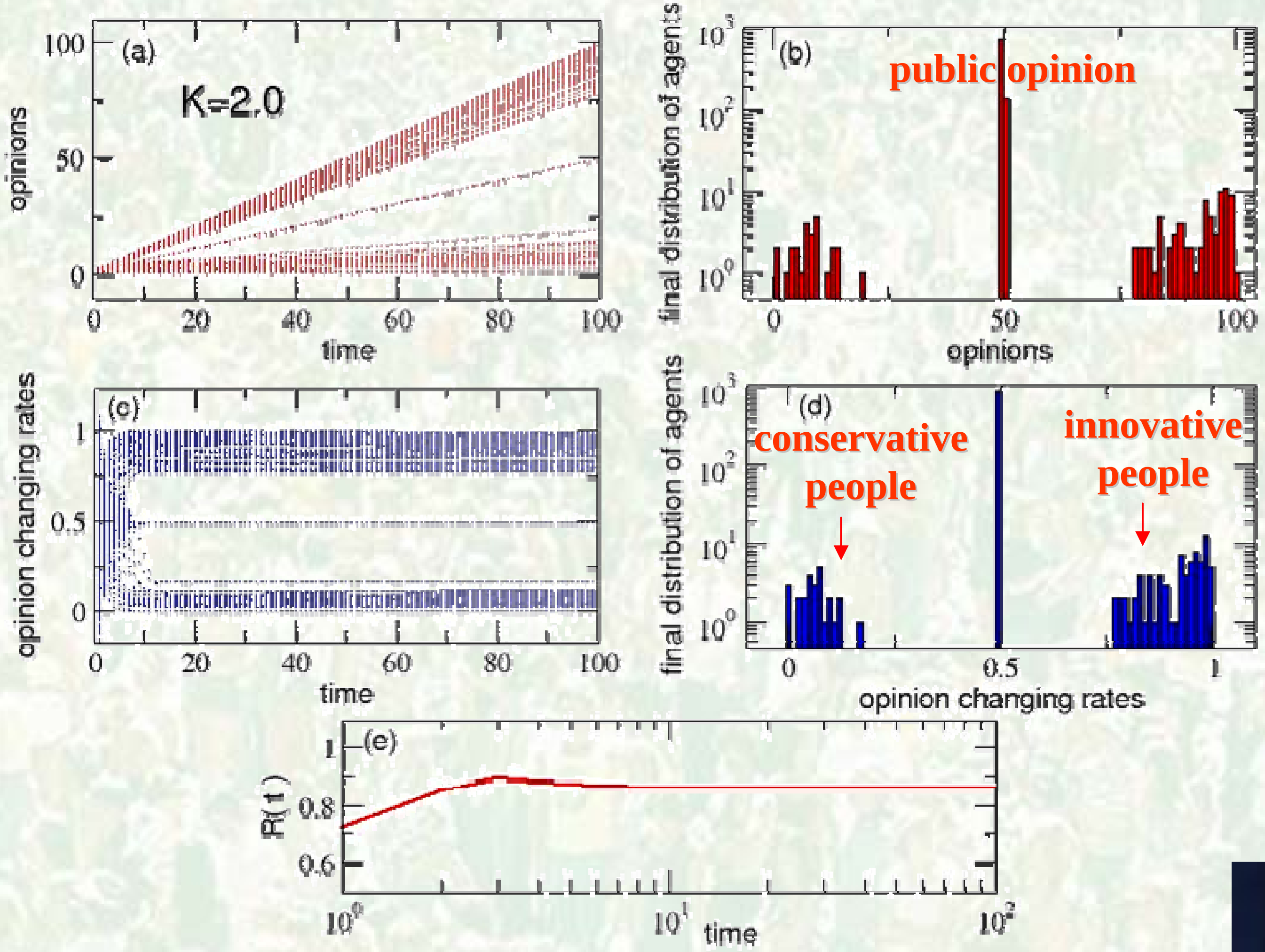
K=1 (incoherent phase) : anarchy



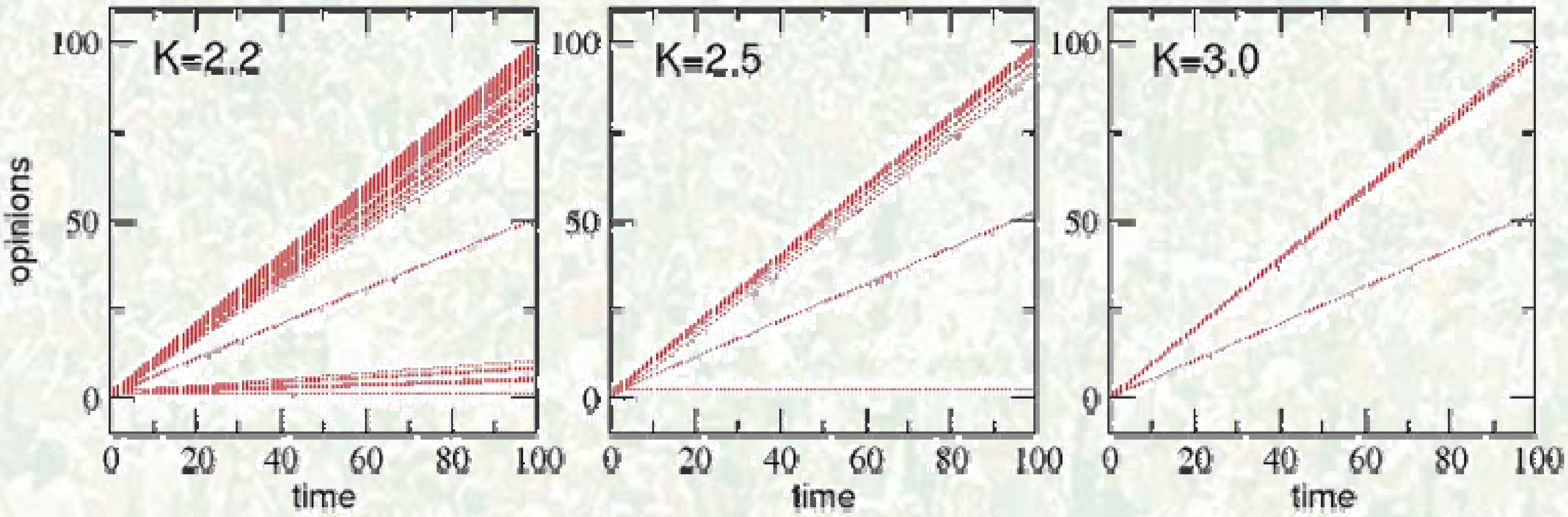
N=1000

K=2 (partially synch. phase) : bipolarism

N=1000

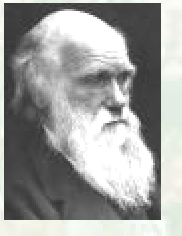


K= 2.2 - 2.5 - 3.0 (partially synchronized phase)

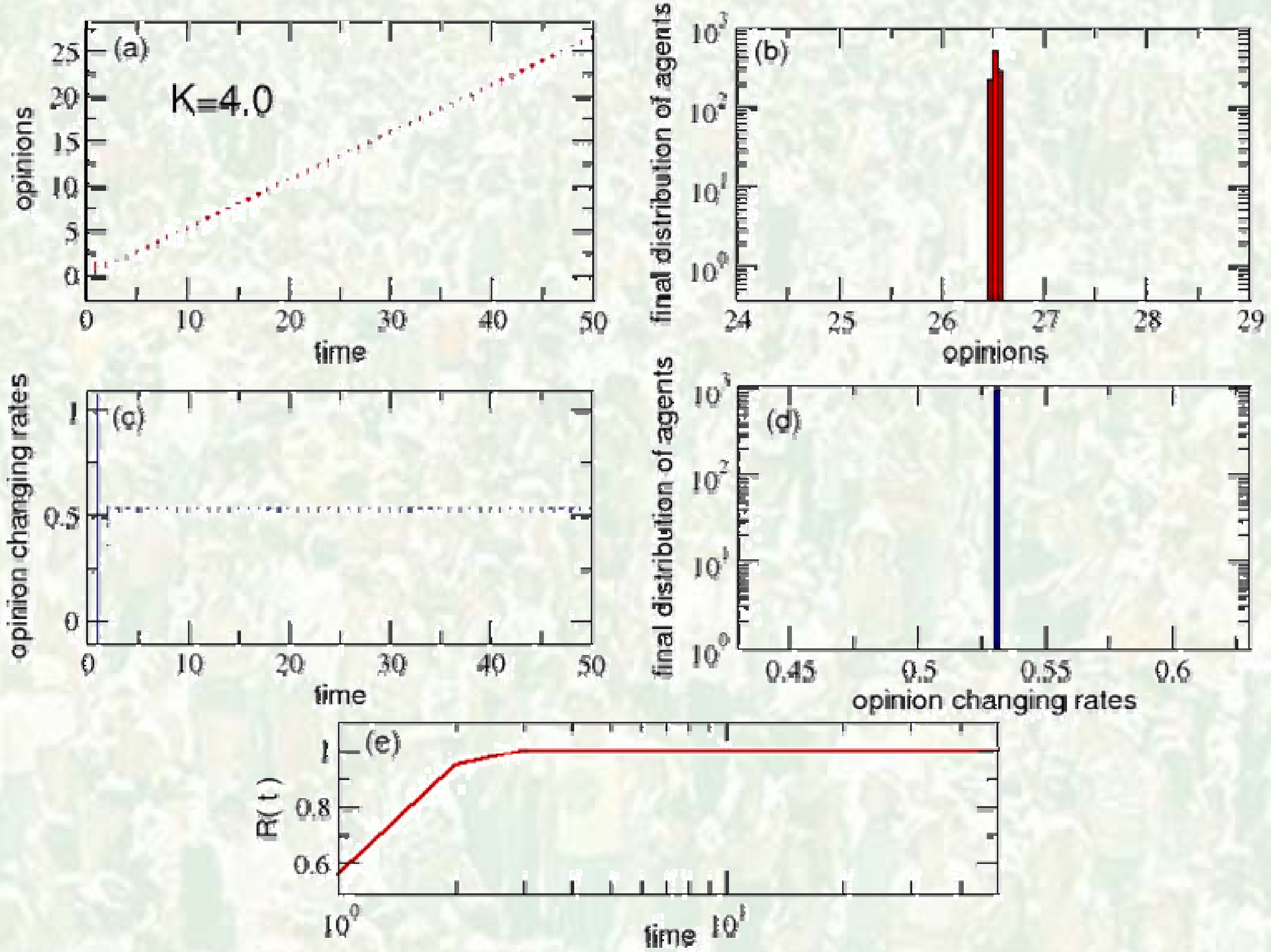


Increasing K in the partially sinchronized phase the innovative group survives longer than the conservative one… **Why?**

"It is not the strongest that survives, nor the most intelligent; it is the one that is the most adaptable to change" *C.Darwin*

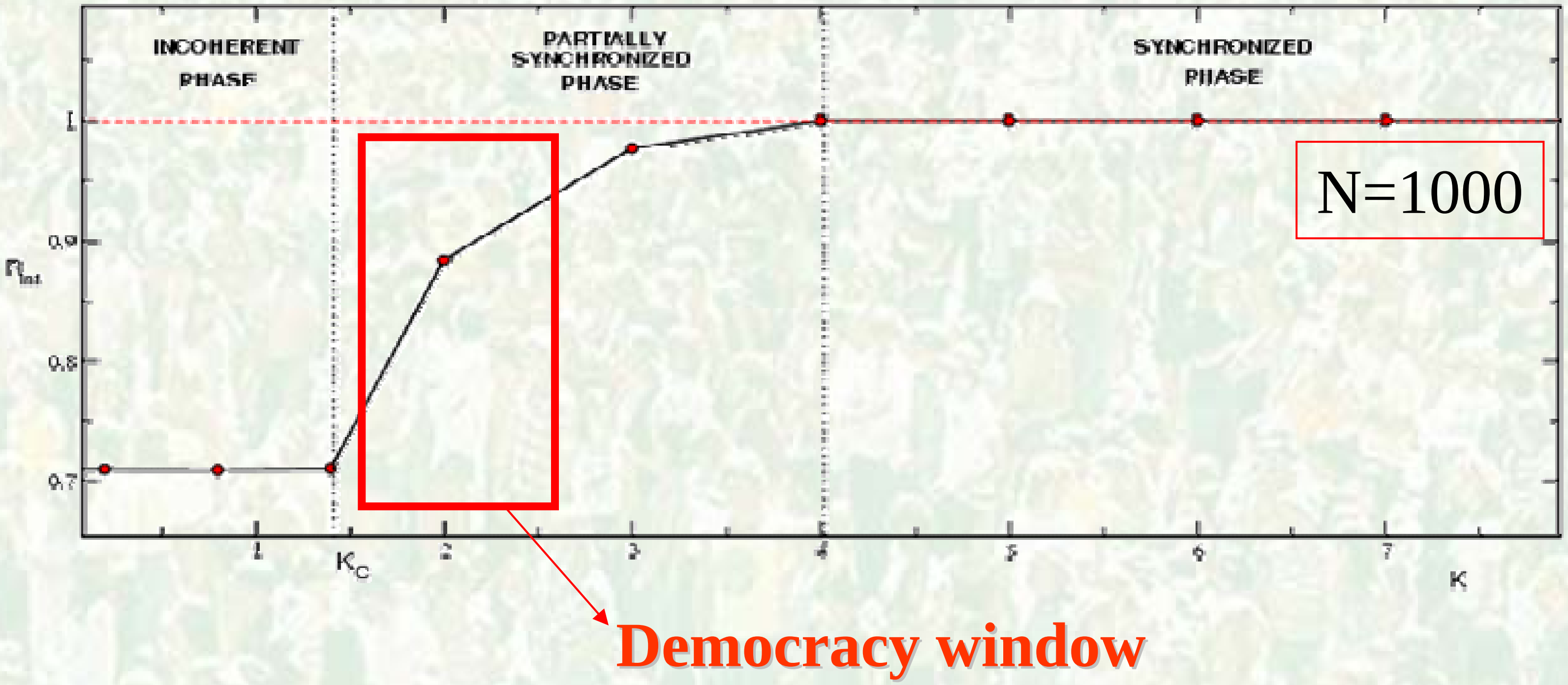


K=4 (synchronized phase) : dictatorship



N=1000

Thus, in order to ensure an equilibrium between conservative and innovative groups (democracy), a changing society needs a coupling K strictly included in a narrow window (1.5<K<2.5)



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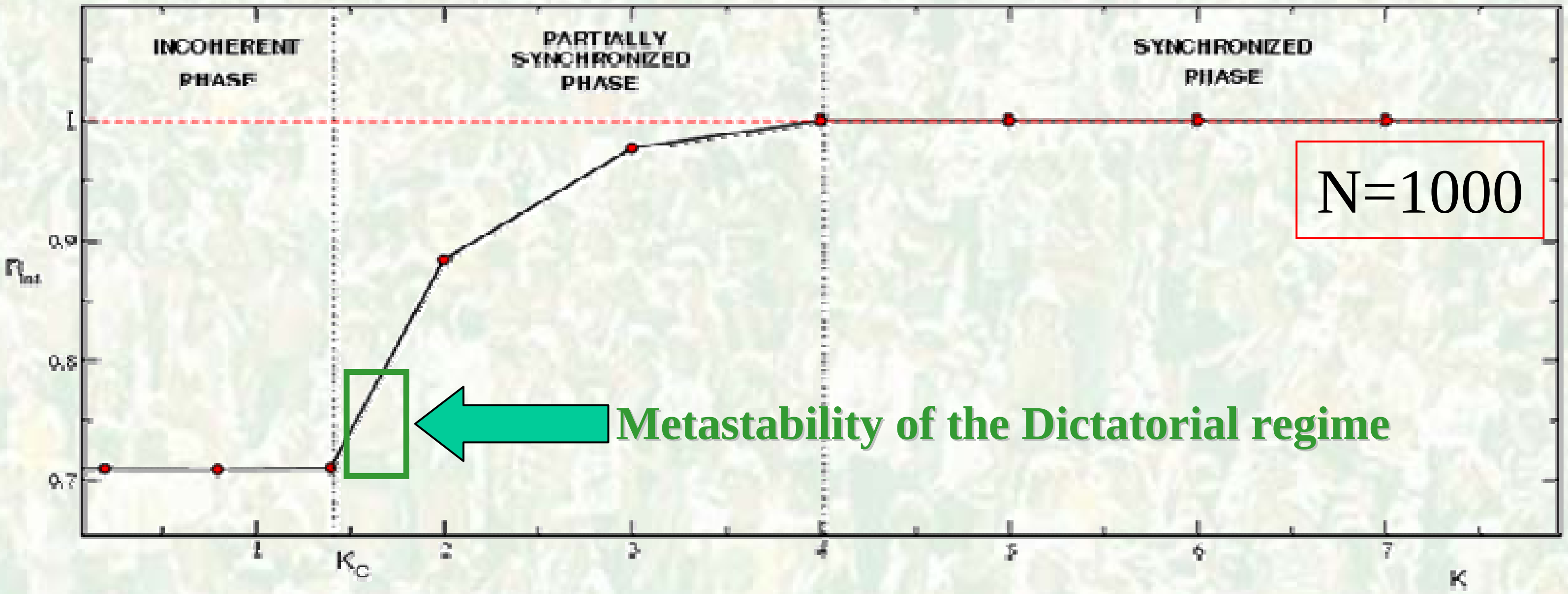
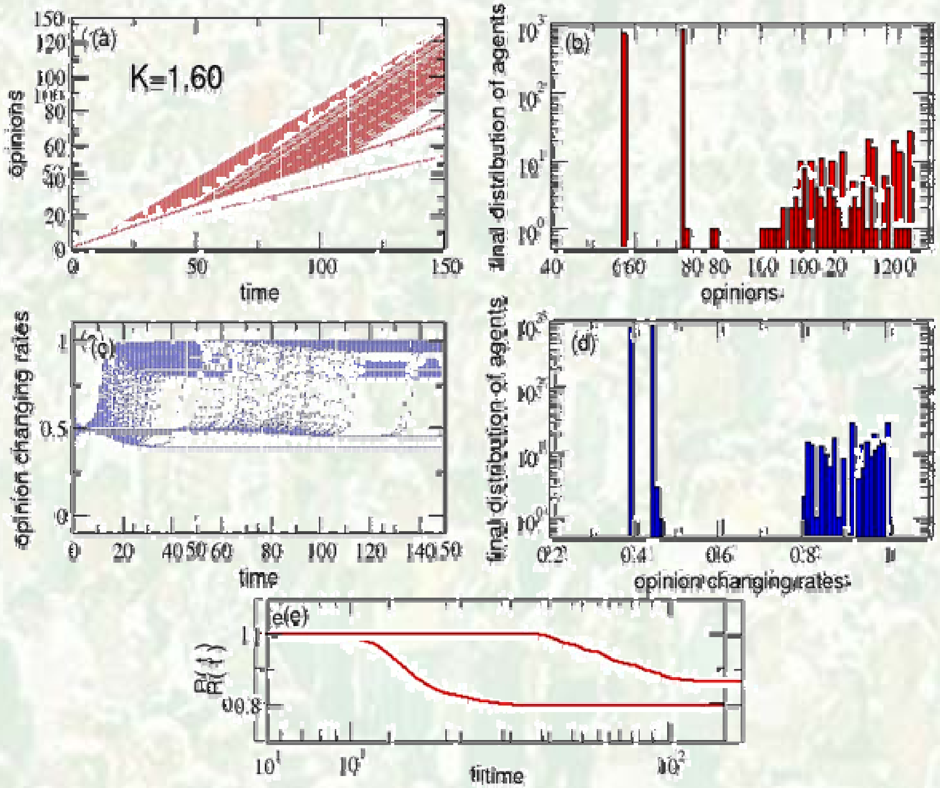


Metastability of the dictatorship regime

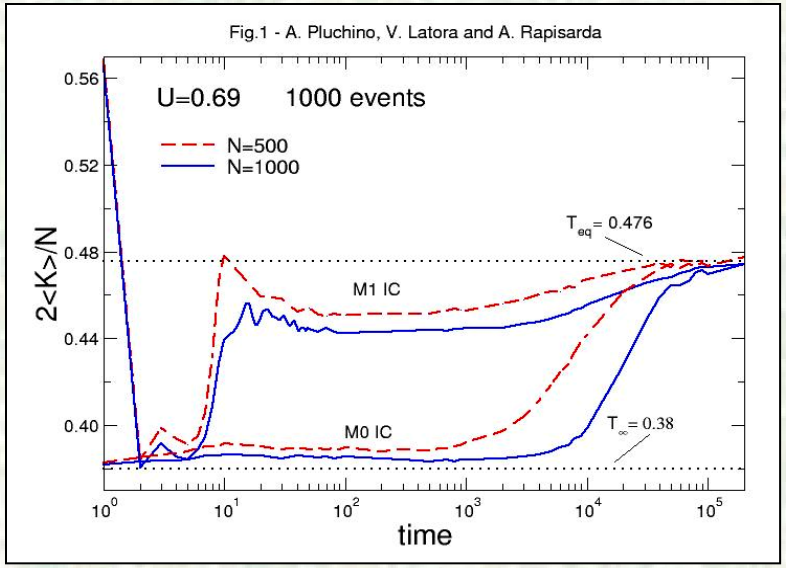
If one starts all the agents with the same opinion (dictatorship) at the beginning of the partially synchronized phase, one observes a metastability regime that becomes stable approaching the value K=1.62



N=1000

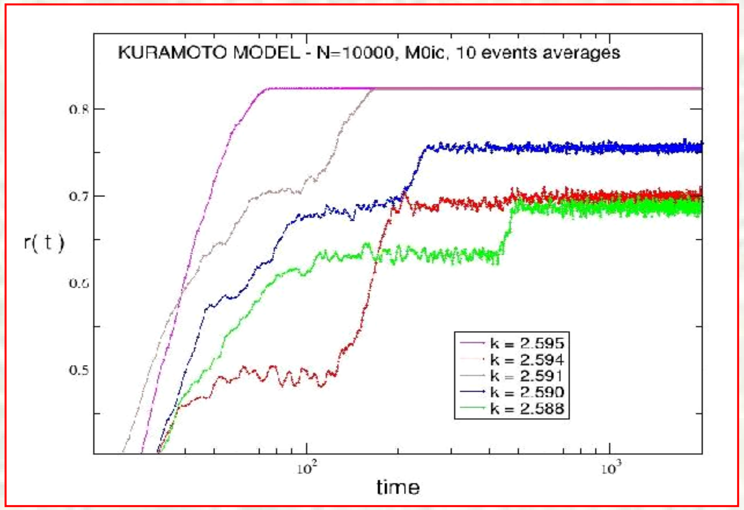
Metastability near the phase transition seems to be ubiquitous in many models:

Hamiltonian Mean Field Model



Pluchino, Latora, Rapisarda, Physica D 193 (2004) 315 ; Physica A 338 (2004) 60

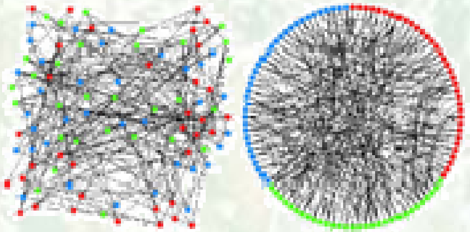
Kuramoto Model



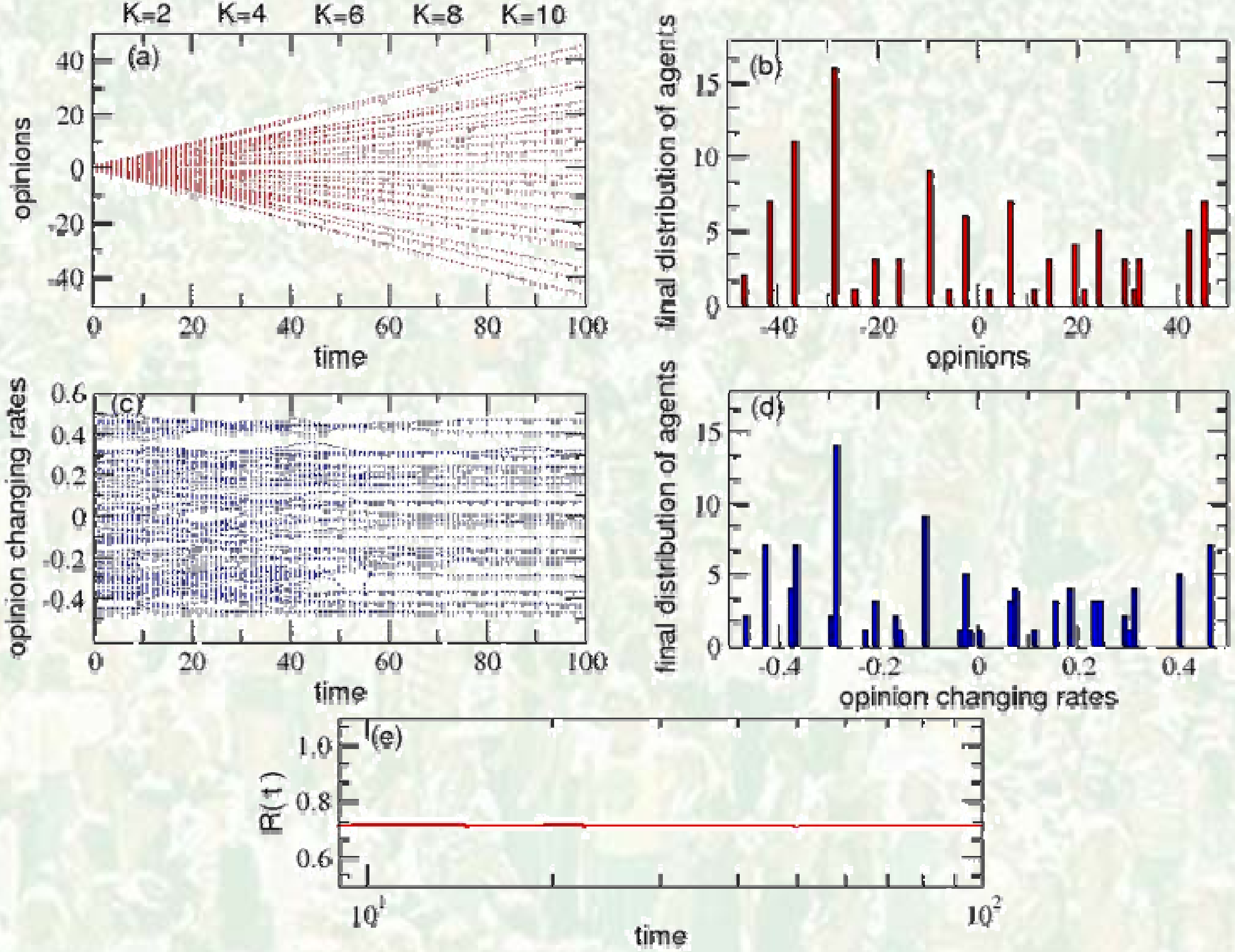
A.Pluchino and A.Rapisarda, Proceedings of the 3rd Int.Conf. NEXT $\Sigma\Phi$ - Kolymbari, Creta (2005)

K-Satisfiability Model

Mezard, Parisi, Zecchina, *'Analityc and Algorithmic Solution of Random Satisfiability Problems'* - Science 279 (2002) p.842



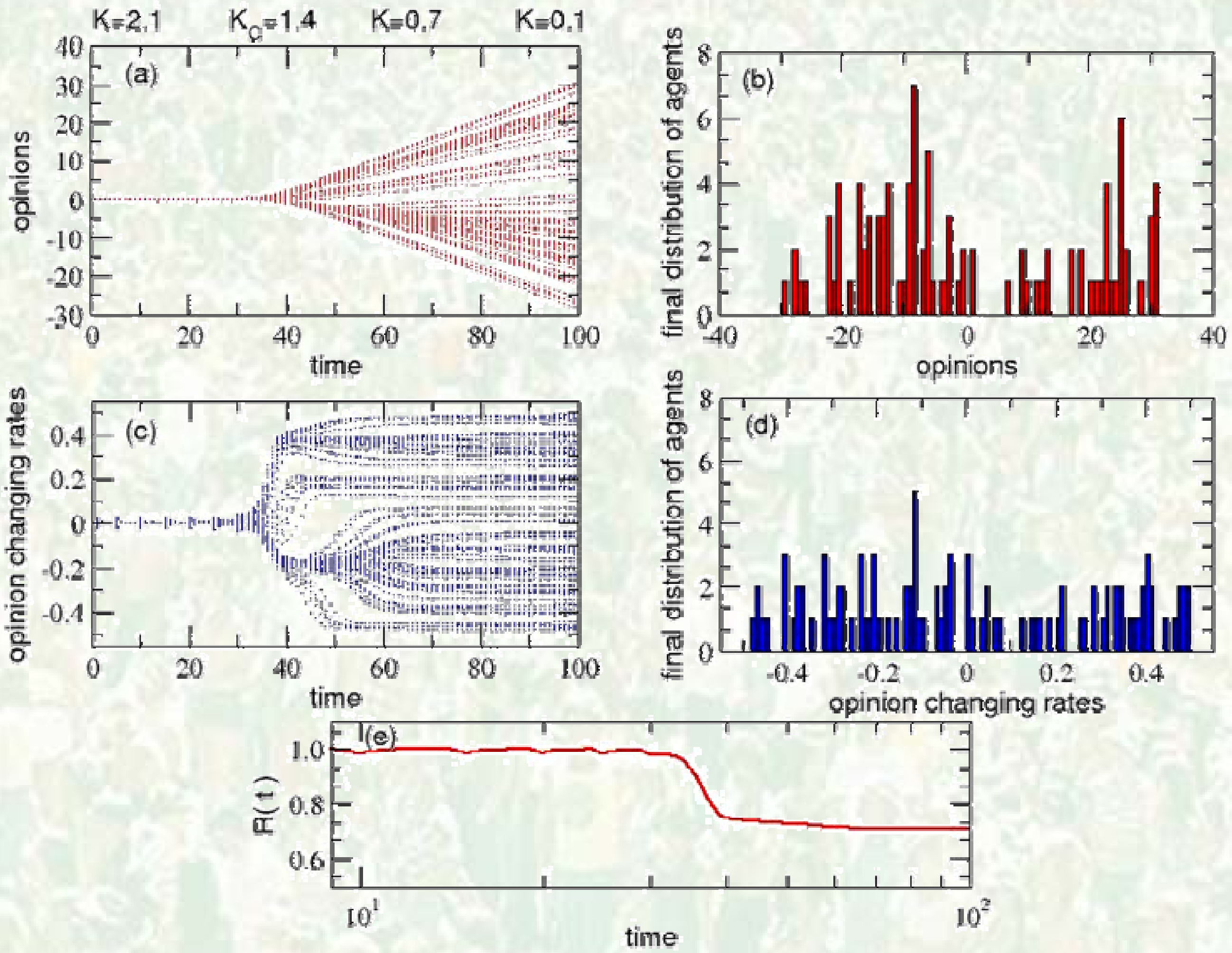
Increasing coupling: from anarchy to democracy



N=100

Political parties
or
opinion clusters
formation

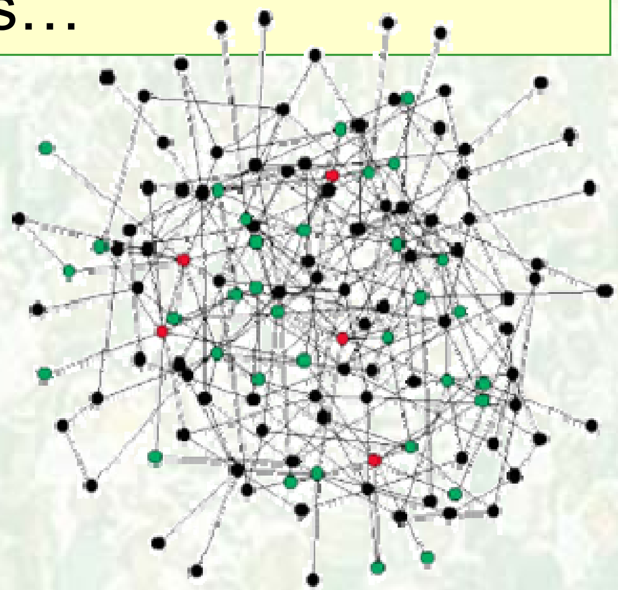
Decreasing coupling: from order to anarchy



N=100

Fall of a dictatorship or dissolution of an empire

More recently we tried to extend the synchronization approach to the problem of **finding community structures** in social networks and in other complex networks…

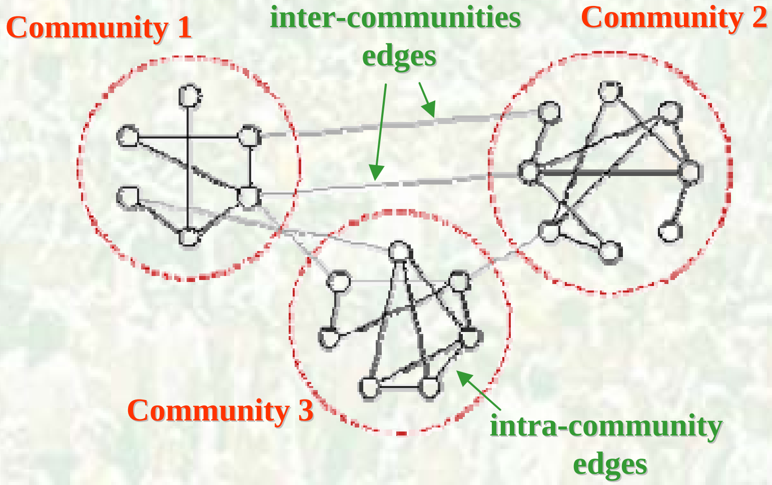


Finding Community Structures in Complex Networks:

Tuning Synchronization in Weighted Networks

Finding Community Structures in Complex Networks

An important open problem in complex networks analysis is the identification of modular structures.



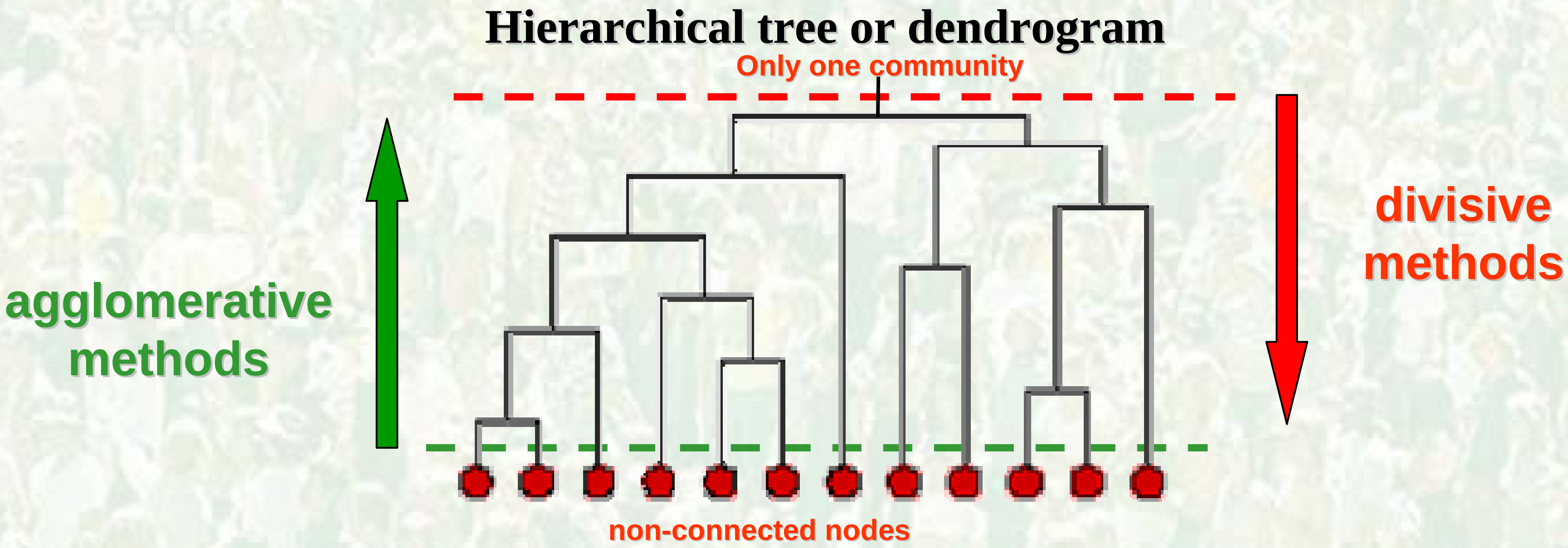
Distinct modules, motives, subgroups or communities within networks can loosely be defined as subset of nodes which are more densely linked, when compared to the rest of the network.

Communities, of course, are fundamental in social networks (parties, cultures, elites), but are also important in biochemical, metabolic or neural networks (functional groups), in the world wide web (thematic clusters), in economic networks, food webs, computer clusters and so on…

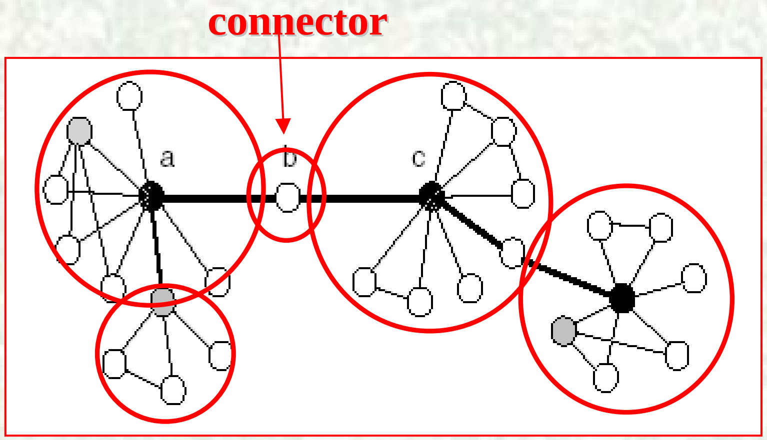
A useful set of techniques for the detection of community structures was firstly developed in social network analysis and is known as **hierarchical clustering methods**…

These techniques are aimed at discovering natural divisions of (social) networks into groups, based on various metric of similarity or strength of connection between vertices.

They fall into two broad classes: **agglomerative** and **divisive** methods, depending on whether they focus on the addition or the removal of edges to or from the network…



Divisive methods remove progressively the edges of the networks in terms of their 'importance', for example the importance in connecting many pairs of nodes (shortest-path betweenness*), or in propagating some information over the network (information centrality**)…



By doing this repeatedly, and recalculating the betweenness at each step, the network breaks iteratively into smaller and smaller components…

…until it breaks into a collection of single non-connected nodes.

The divisive algorithm produces a hierarchy of subdivisions of the network. But how to know which subdivision is the best one for a given network?

Clearly we need some measure of the cohesiveness of the communities…

This measure is the "modularity" Q*, a quantity that, at each step, compares the actual fraction of edges intra-community with the expected value in the same network with random connections, and allows us to test if the communities found by the divisive algorithm are the good ones…

*M.E.J.Newman and M.Girvan, 2004 *Phys. Rev. E* **69** 026113

S.Fortunato, V.Latora, M.Marchiori, 2004 *Phys. Rev. E* **70 056104

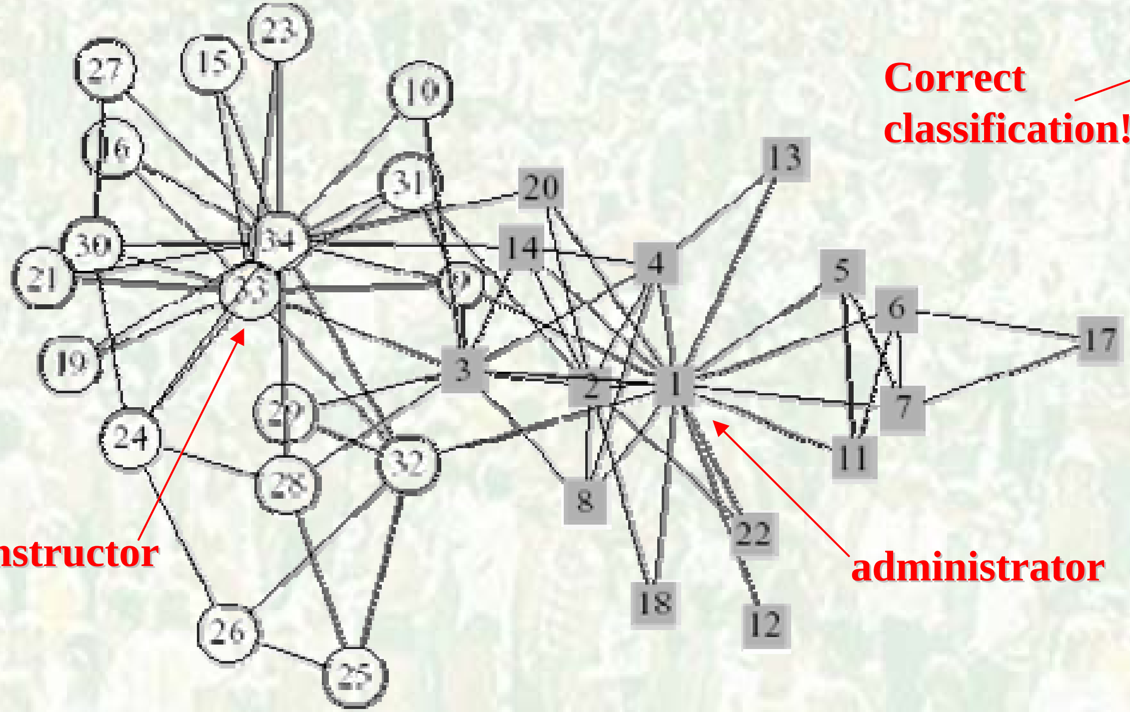
Zachary's Karate Club friendships network

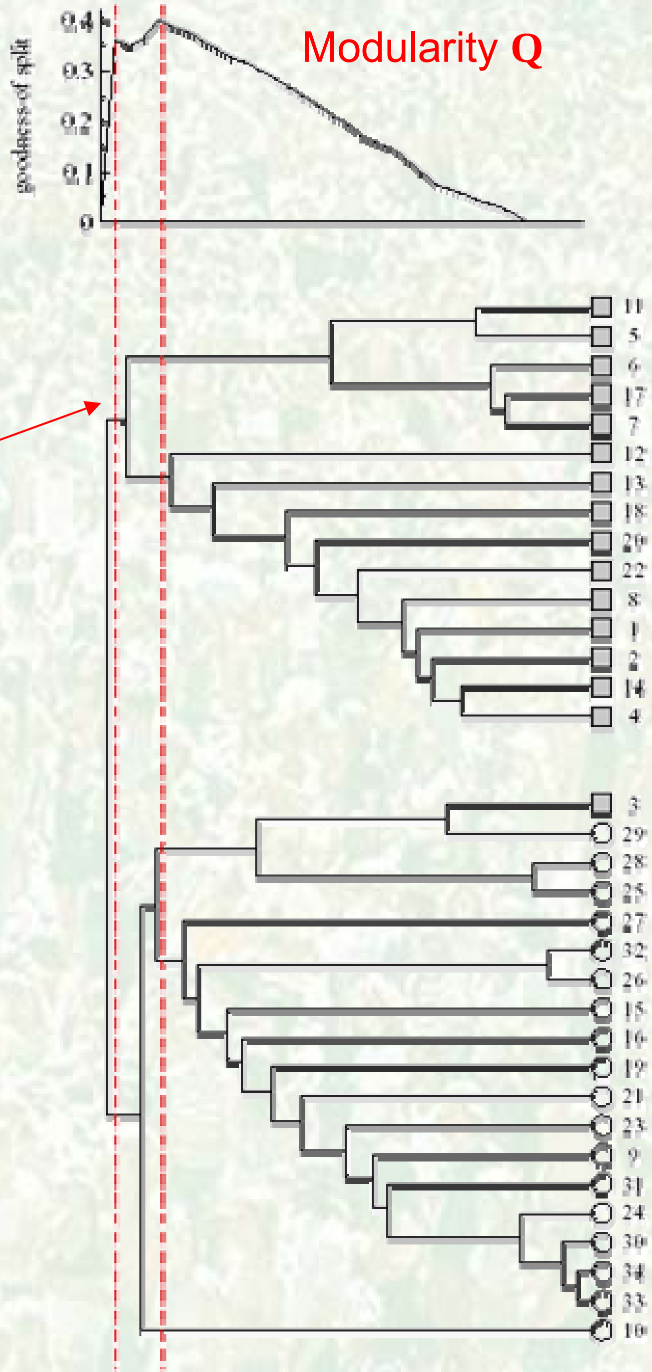
$$Q = Tr\,\mathrm{e} - \left\| \mathrm{e}^2 \right\|$$

with

$$\mathrm{e} = \left\{ \mathrm{e}_{ij} \right\}_{n_c \times n_c}$$

Modularity **Q**





Correct classification!

instructor

administrator

Community 2 (18 nodes) **Community 1 (16 nodes)**

Shortest-path betweenness method

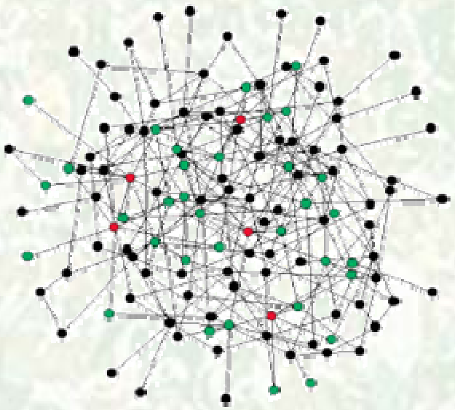
M.E.J.Newman and M.Girvan, 2004 *Phys. Rev. E* **69** 026113

How synchronization can be useful for the identification of community structures in complex networks?

THE MASTER STABILITY FUNCTION APPROACH TO ENHANCE SYNCHRONIZATION IN COMPLEX NETWORKS

Suppose to have a (unweighted, undirected) network of N linearly coupled identical oscillators*. The equation of motion reads:

Network with N nodes



coupling strenght

$$\underbrace{\dot{\vec{x}}_i = \vec{F}(\dot{\vec{x}}_i)}_{\text{dynamical system defined over each node of the network}} - \sigma \sum_{j=1}^{N} G_{ij}\, \vec{H}[\vec{x}_i - \vec{x}_j] \qquad i = 1,\ldots,N$$

coupling matrix

vectorial coupling function

If G has a real spectrum of eigenvalues λ_i (i.e. for symmetric coupling) and we associate λ_1 to the state $\mathbf{x}_s(t)$, the stability of the synchronous manifold ($\mathbf{x}_i(t)=\mathbf{x}_s(t)$, $\forall$ i) requires that all the conditional Lyapunov exponents Λ associated with $\lambda_2 \leq \ldots \leq \lambda_i \leq \ldots \leq \lambda_N$ would be negative.

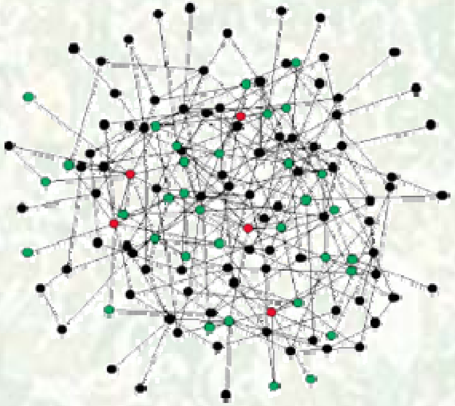
Defining the Master Stability Function (MSF) as the largest Lyapunov exponent Λ_{max} versus a parameter $\nu=\sigma\lambda$, it can be shown* that, for a large class of oscillatory systems, the MSF is negative in a finite parameter interval (ν_1, ν_2).

*M.Chavez, D.U.Hwang, A.Amann, H.G.E.Hentschel and S.Boccaletti, *Phys. Rev. Lett.* **94** 218701 (2005)

THE MASTER STABILITY FUNCTION APPROACH TO ENHANCE SYNCHRONIZATION IN COMPLEX NETWORKS

Suppose to have a (unweighted, undirected) network of N linearly coupled identical oscillators*. The equation of motion reads:

Network with N nodes



coupling strenght

$$\underbrace{\dot{\vec{x}}_i = \vec{F}(\dot{\vec{x}}_i)}_{\text{dynamical system defined over each node of the network}} - \sigma \sum_{j=1}^{N} G_{ij}\, \vec{H}[\vec{x}_i - \vec{x}_j] \qquad i = 1, \ldots, N$$

coupling matrix

vectorial coupling function

If G has a real spectrum of eigenvalues λ_i (i.e. for symmetric coupling) and we associate λ_1 to the state $\mathbf{x}_s(t)$, the stability of the synchronous manifold ($\mathbf{x}_i(t)=\mathbf{x}_s(t)$, $\forall$ i) requires that all the conditional Lyapunov exponents Λ associated with $\lambda_2 \leq \ldots \leq \lambda_i \leq \ldots \leq \lambda_N$ would be negative.

Thus the condition for synchronization stability is governed by the ratio λ_N/λ_2: the more packed the eigenvalues of G are, the higher is the chance of having all Lyapunov exponents into the stability range for some σ.

*M.Chavez, D.U.Hwang, A.Amann, H.G.E.Hentschel and S.Boccaletti, *Phys. Rev. Lett.* **94** 218701 (2005)

At this point one can use the master stability function approach:

1) to find the best synchronization condition of a given network*
2) to tune the synchronization of a network in order to identify community structures**

Both the results can be realized with an opportune choice of the coupling matrix G_{ij} in the network equation, by means of a weighting procedure that assignes to each edge a 'load' l_{ij} equal to its betweenness (i.e. the number of shortest paths that are making use of that edge):

coupling matrix G=G(α)

$$\dot{\vec{x}}_i = \vec{F}(\dot{\vec{x}}_i) - \sigma \sum_{j \in K_i} \left(\frac{l_{ij}{}^{\alpha}}{\sum_{j \in K_i} l_{ij}{}^{\alpha}} \right) \vec{H}[\vec{x}_i - \vec{x}_j] \qquad i = 1, \ldots, N$$

where α is a real tunable parameter and K_i is the set of neighbors of the i^{th} node.

master stability function arguments apply

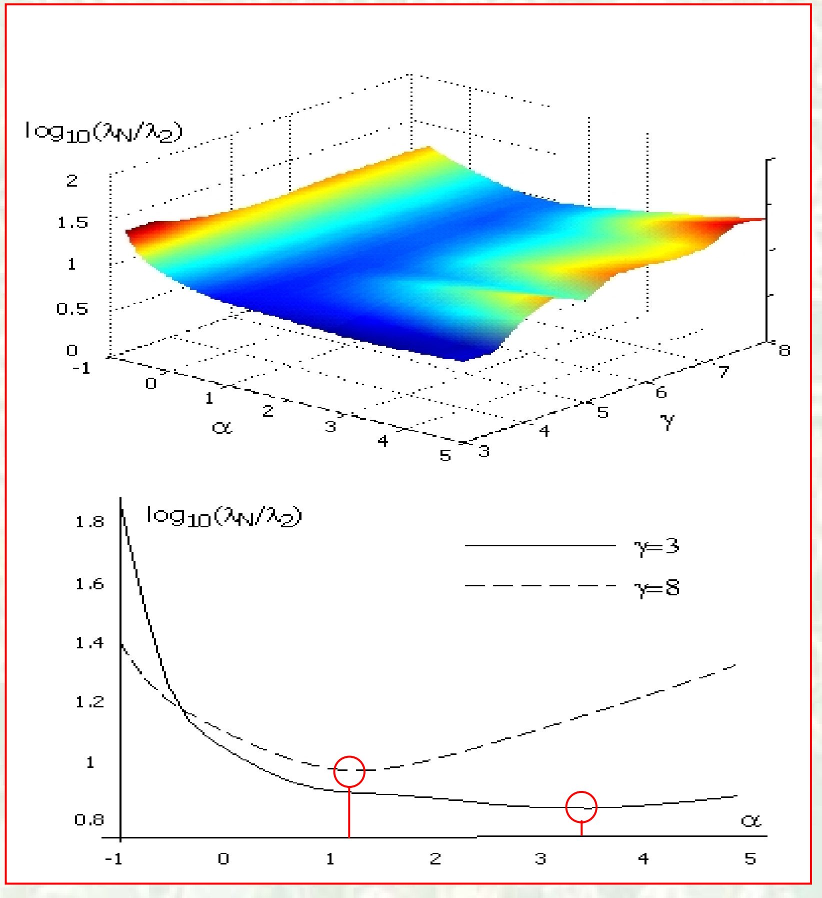
*M.Chavez, D.U.Hwang, A.Amann, H.G.E.Hentschel and S.Boccaletti, *Phys. Rev. Lett.* **94** 218701 (2005)

**A.Pluchino, V.Latora, A.Rapisarda and S.Boccaletti, *in preparation*

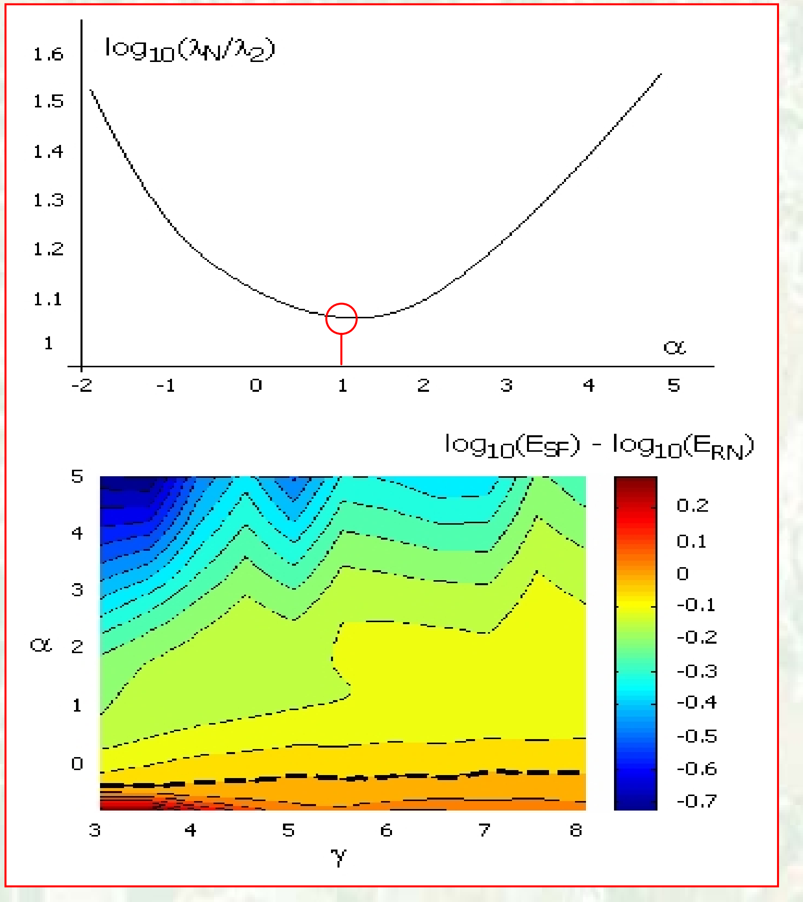
Finding the best synchronization condition for a network of oscillators

$$\dot{\vec{x}}_i = \vec{F}(\dot{\vec{x}}_i) - \frac{\sigma}{\sum_{j\in K_i} l_{ij}^{\ \alpha}} \sum_{j\in K_i} l_{ij}^{\ \alpha}\, \vec{H}[\vec{x}_i - \vec{x}_j] \qquad i = 1,\ldots,N$$

Scale free networks



Random networks



M.Chavez, D.U.Hwang, A.Amann, H.G.E.Hentschel and S.Boccaletti, *Phys. Rev. Lett.* **94** 218701 (2005)

Tuning the synchronization of a network of oscillators and finding community structures

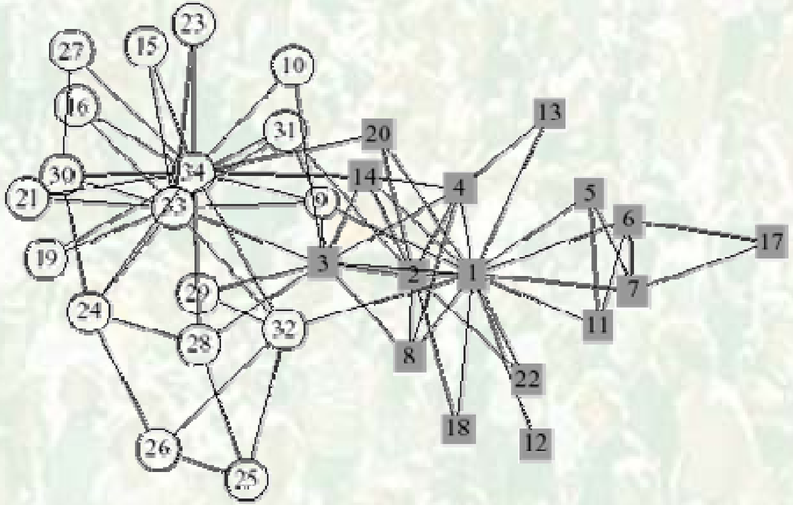
$$\dot{\vec{x}}_i = \vec{F}(\vec{x}_i) - \frac{\sigma}{\sum_{j \in K_i} l_{ij}^{\alpha}} \sum_{j \in K_i} l_{ij}^{\alpha} \vec{H}[\vec{x}_i - \vec{x}_j] \qquad i = 1,\dots,N$$

$\alpha \to -\infty$ **Edges with the greatest betweenness are weighted less and less and oscillators should progressively desynchronize…**

Kuramoto's non identical 1D oscillators

$$\dot{\vartheta}_i = \omega_i + \frac{K}{\sum_{j \in K_i} l_{ij}^{\alpha}} \sum_{j \in K_i} l_{ij}^{\alpha} \sin(\theta_j - \theta_i) \qquad i = 1,\dots,N$$

Zachary's network (N=34)



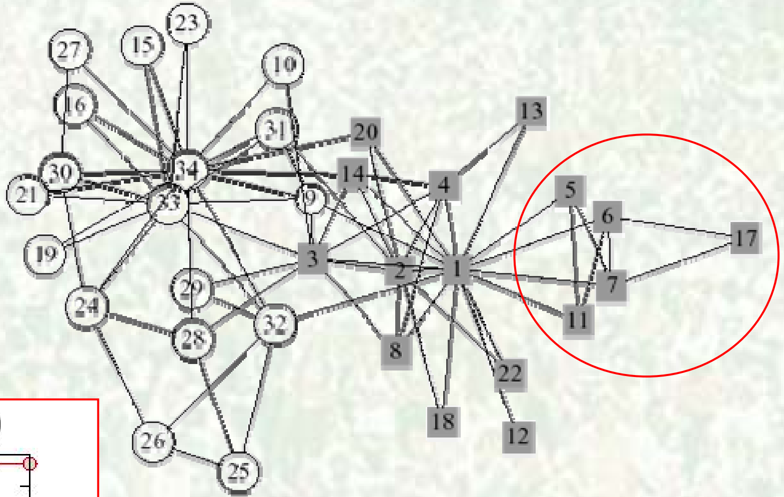
Chaotic Rössler identical 3D oscillators

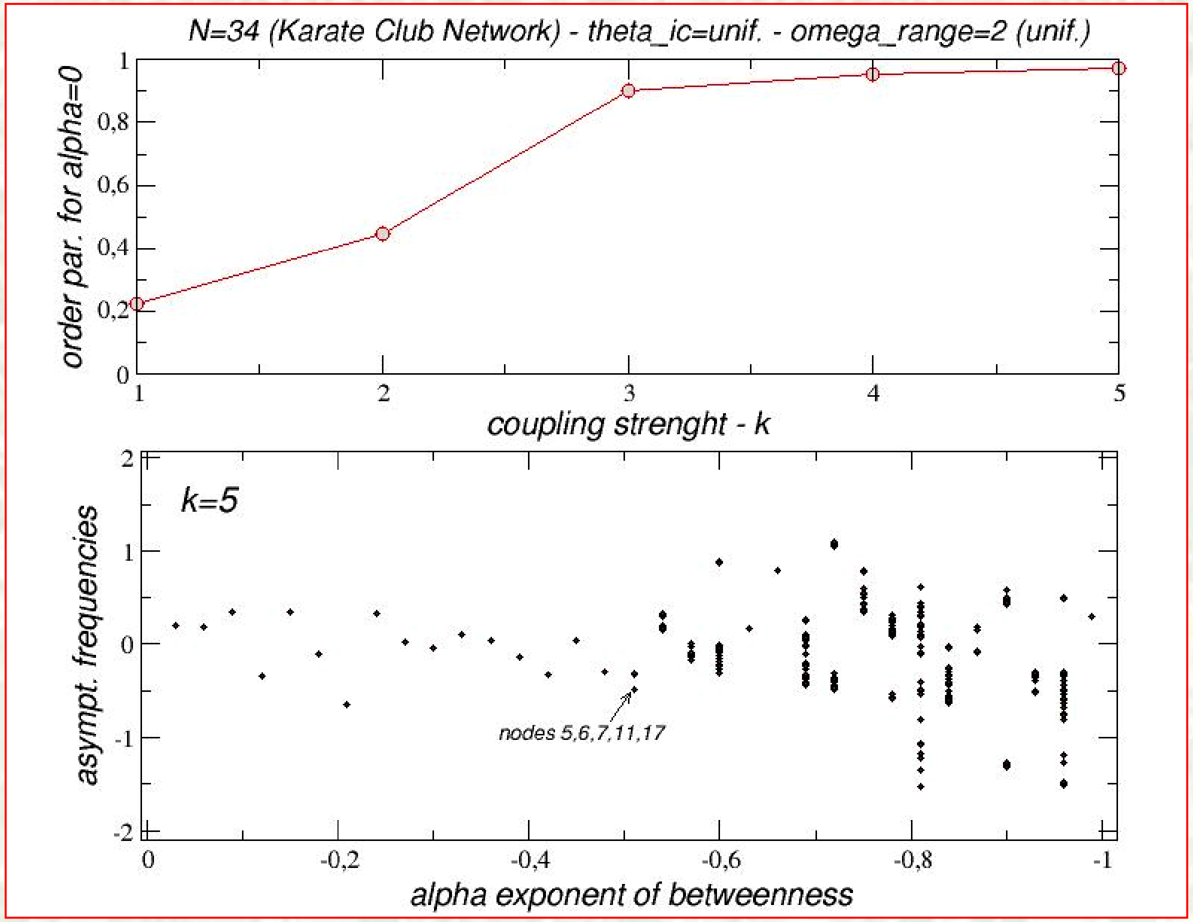
$$\begin{cases} \dot{x}_i = -\omega y_i - z_i - \dfrac{K}{\sum_{j \in K_i} l_{ij}^{\alpha}} \sum_{j \in K_i} l_{ij}^{\alpha} (x_i - x_j) \\ \dot{y}_i = \omega x_i + 0.165 y_i \\ \dot{z}_i = 0.2 + z_i (x_i - 10) \end{cases} \qquad i = 1,\dots,N$$

A.Pluchino, V.Latora, A.Rapisarda and S.Boccaletti, *in preparation*

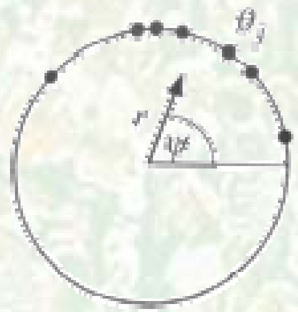
Kuramoto's non identical 1D oscillators

$$\dot{\vartheta}_i = \omega_i + \frac{K}{\sum_{j\in K_i} l_{ij}^{\alpha}} \sum_{j\in K_i} l_{ij}^{\alpha} \sin(\theta_j - \theta_i) \qquad i = 1,\dots,N$$





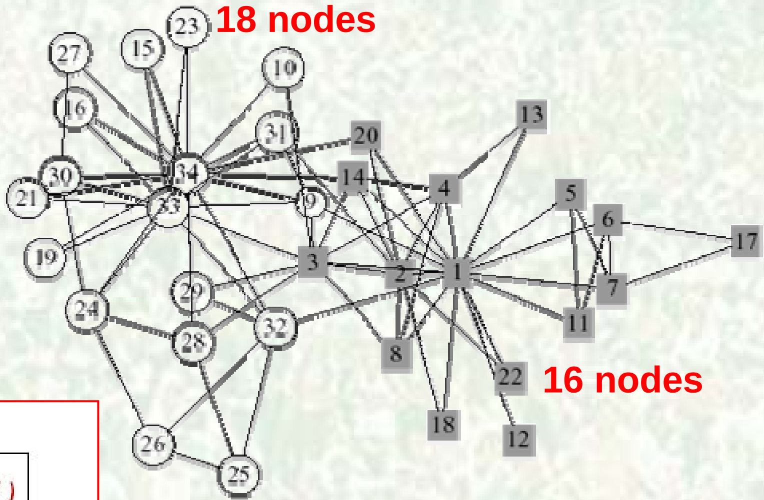
$$r_{\text{inf}} = \left\langle \frac{1}{N} \left| \sum_{i=1}^{N} e^{j\vartheta_i(t)} \right| \right\rangle_{t\to\infty}$$

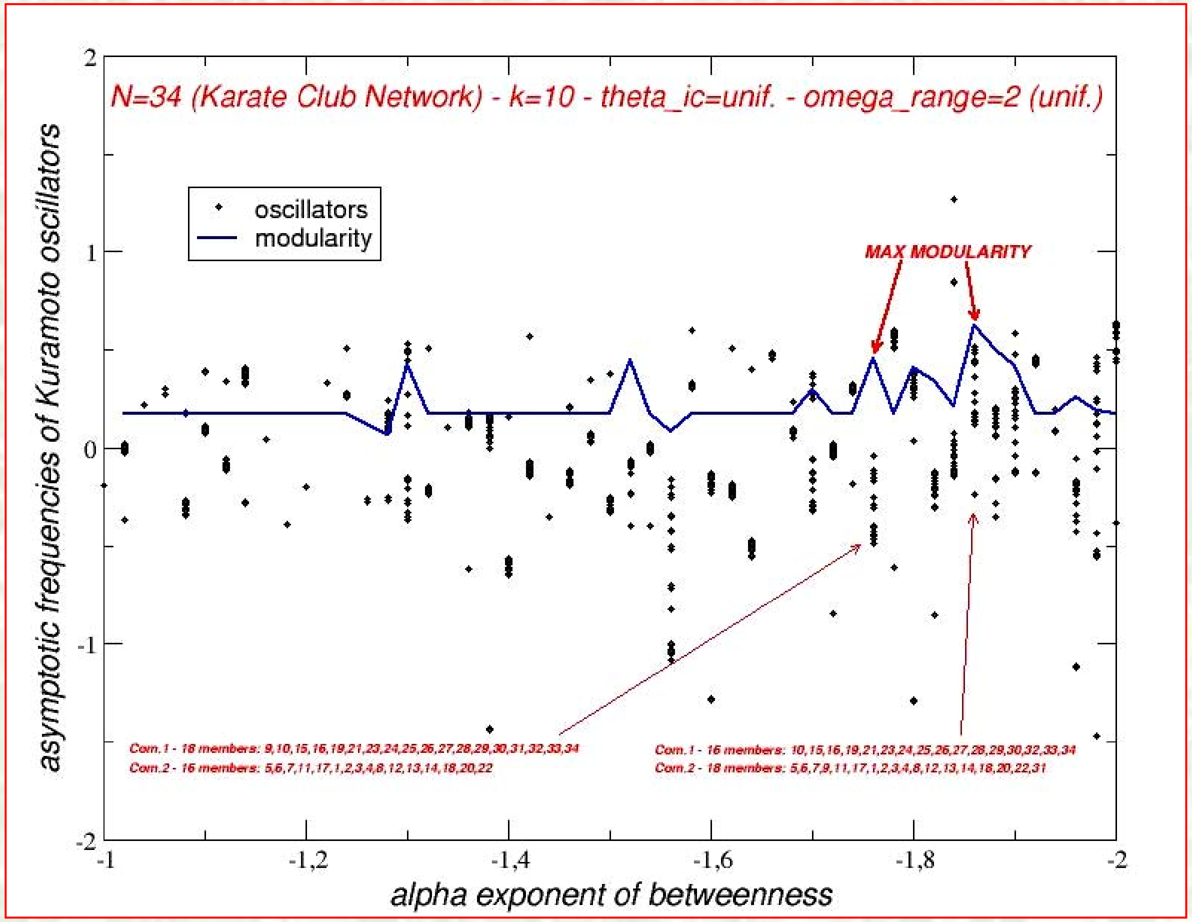


A.Pluchino, V.Latora, A.Rapisarda and S.Boccaletti, *in preparation*

Kuramoto's non identical 1D oscillators

$$\dot{\vartheta}_i = \omega_i + \frac{K}{\sum_{j \in K_i} l_{ij}^{\alpha}} \sum_{j \in K_i} l_{ij}^{\alpha} \sin(\theta_j - \theta_i) \qquad i = 1, \dots, N$$





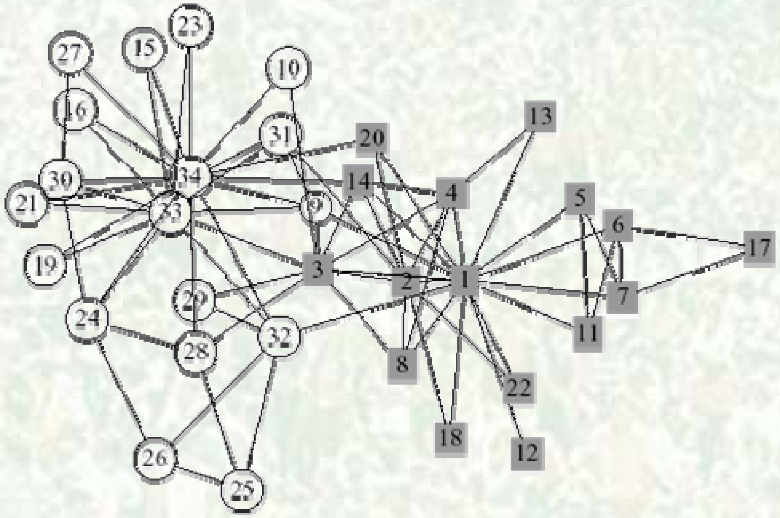
The two main communities of Zachary's network have been almost correctly recognized

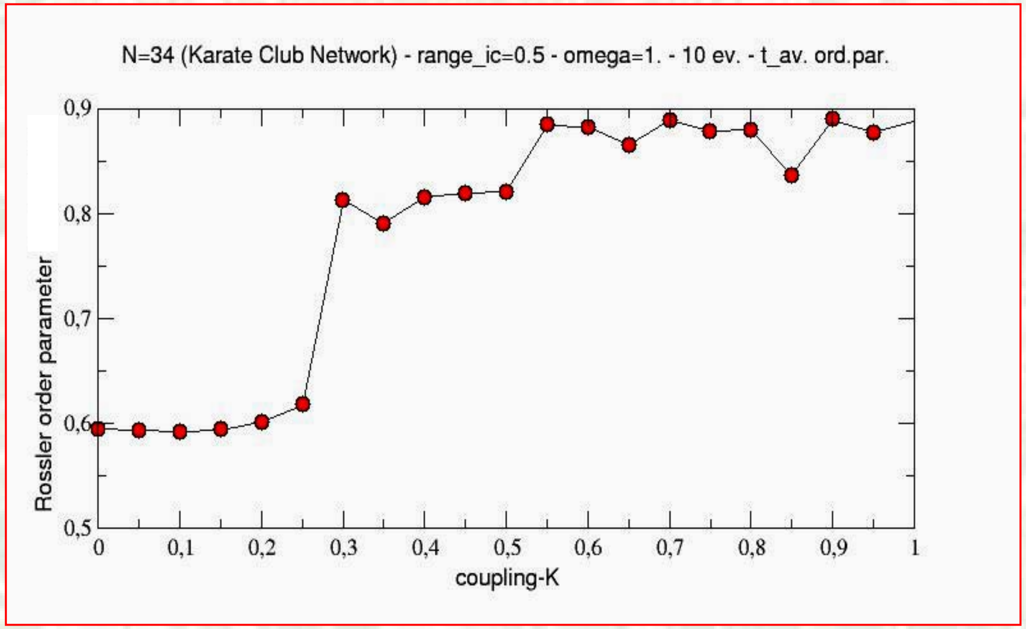
A.Pluchino, V.Latora, A.Rapisarda and S.Boccaletti, *in preparation*

Chaotic Rössler identical 3D oscillators

$$
\begin{cases}
\dot{x}_i = -\omega y_i - z_i - \dfrac{K}{\sum_{j\in K_i} l_{ij}^{\ \alpha}} \sum_{j\in K_i} l_{ij}^{\ \alpha}(x_i - x_j) \\
\dot{y}_i = \omega x_i + 0.165 y_i \\
\dot{z}_i = 0.2 + z_i(x_i - 10)
\end{cases}
$$

$$i = 1, \ldots, N$$





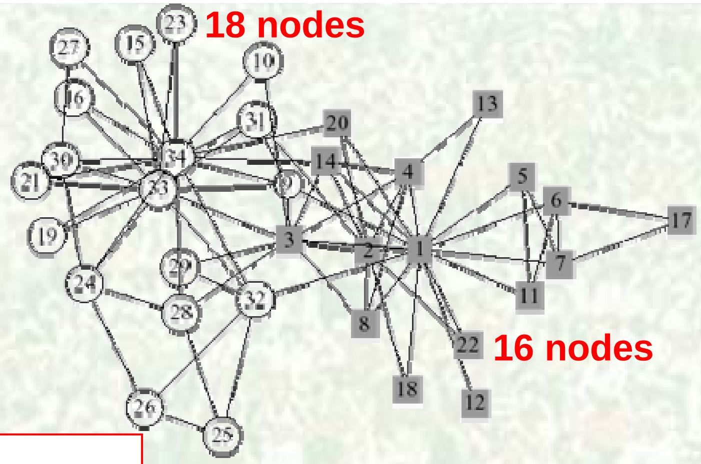
$$\psi_{\text{inf}} = \left\langle \frac{1}{N} \left| \sum_{i=1}^{N} e^{j\Phi i(t)} \right| \right\rangle_{t\to\infty}$$

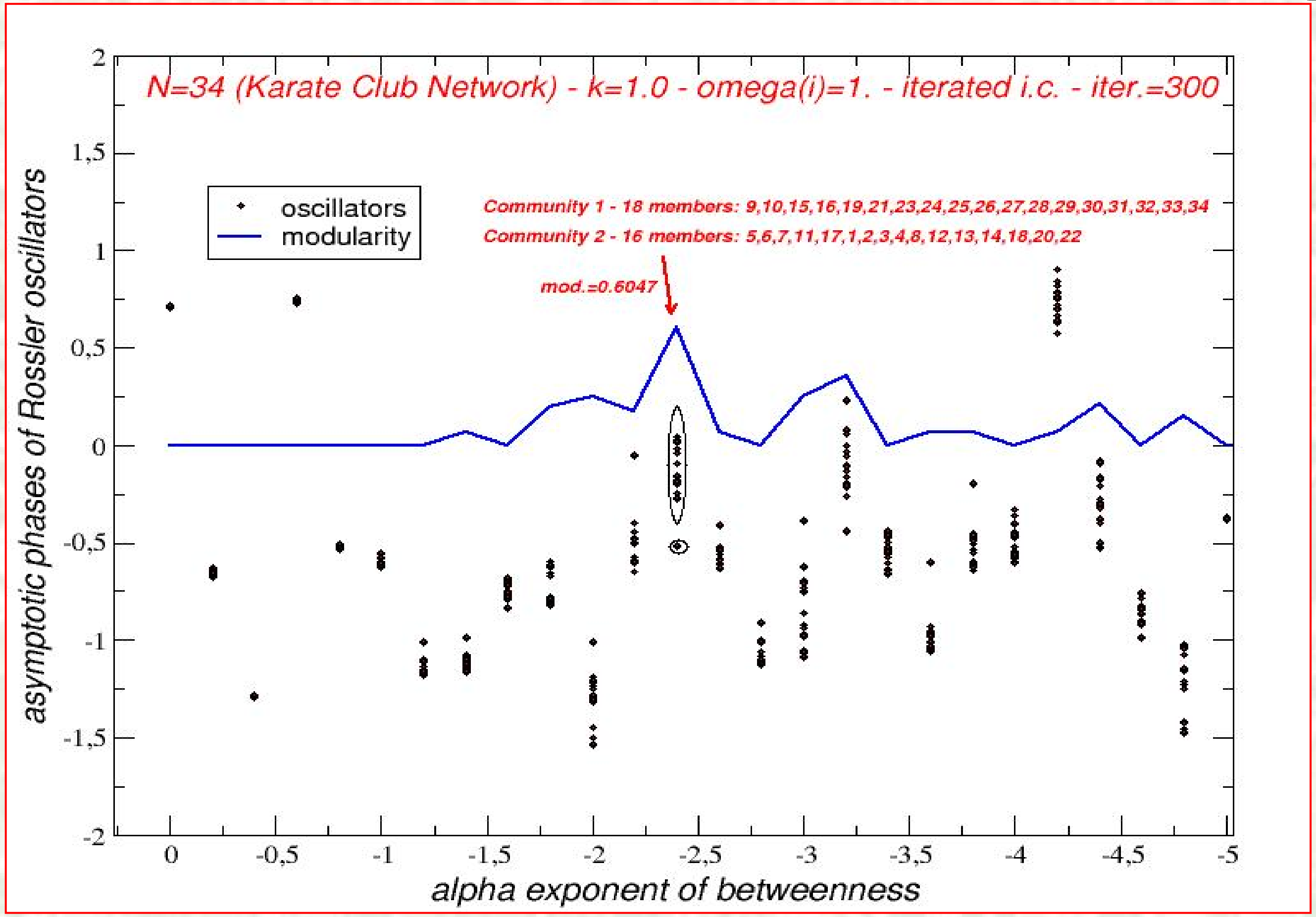
$$\phi_i(t) = \arctan\left[\frac{y_i(t)}{x_i(t)}\right]$$

A.Pluchino, V.Latora, A.Rapisarda and S.Boccaletti, *in preparation*

Chaotic Rössler identical 3D oscillators

$$
\begin{cases}
\dot{x}_i = -\omega y_i - z_i - \dfrac{K}{\sum_{j\in K_i} l_{ij}^{\ \alpha}} \sum_{j\in K_i} l_{ij}^{\ \alpha}(x_i - x_j) \\
\dot{y}_i = \omega x_i + 0.165 y_i \\
\dot{z}_i = 0.2 + z_i(x_i - 10)
\end{cases}
\qquad i = 1,\ldots,N
$$





In this case the two main communities of Zachary's network have been perfectly recognized

A.Pluchino, V.Latora, A.Rapisarda and S.Boccaletti, *in preparation*

Work in progress: further tests of our method…

- sensitivity tests with ad hoc networks with a well known community structure

- sensitivity tests with larger real networks with various topologies (scale free, random, small world…)

- comparison of the computational performance of our method with those of other community identification methods

- testing the method using other dynamical systems, like for example the OCR model on different topologies

So, we conclude that surely **synchronization** can play an important role in sociophysics, in opinion dynamics and, more in general, in complex networks dynamics...

...but are not excluded interesting applications also to political science...☺

Thanks for the attention!

Chaos And Complexity Theoretical University Study

CACTUS Group Catania

http://www.ct.infn.it/~cactus

'FACE' SYNCHRONIZATION