A.Pluchino[a], A.Rapisarda[a], C.Tsallis[b,c]

Noise, Synchrony and Correlations at the Edge of Chaos

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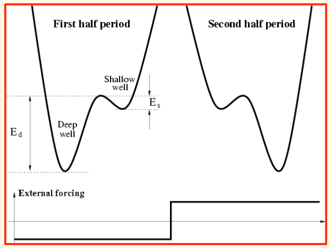
c) **Santa Fe Institute** - Santa Fe, NM 87501, USA

Outline

- THE BENEFICIAL ROLE OF NOISE IN PHYSICAL, BIOLOGICAL AND SOCIAL SYSTEMS
- PATTERNS OF SYNCHRONIZATION IN SYSTEMS OF CHAOTIC LOGISTIC MAPS (CML MODEL)
- NOISE INDUCED LONG-RANGE CORRELATIONS AT THE EDGE OF CHAOS
- Q-STATISTICS STUDY OF TWO-TIME RETURNS: CORRELATIONS AND MEMORY EFFECTS
- ANALOGIES WITH FINANCIAL ANALYSIS OF INTEROCCURENCE TIMES AND ACF

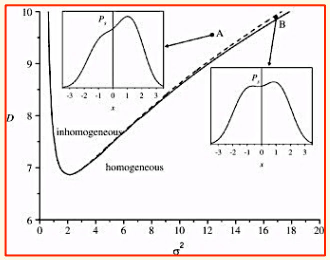
The beneficial role of noise in physics and biology



STOCASTIC RESONANCE

A system embedded in a noisy environment acquires an enhanced sensitivity towards small external time-dependent forcing.

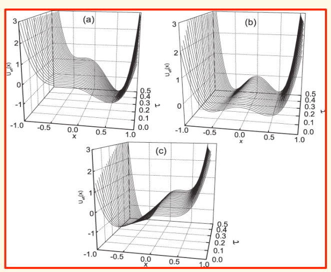
- R. Benzi, G. Parisi, A. Sutera, A. Vulpiani, Tellus 34, 10 (1982)
- L. Gammaitoni, P. H¨anggi, P. Jung and F. Marchesoni, Rev. Mod. Phys. 70, 1 (1998)



NOISE INDUCED NON-EQUILIBRIUM PHASE TRANSITIONS

Noise generates an ordered symmetry-breaking state through a genuine second-order phase transition, whereas no such transition is observed in the absence of noise.

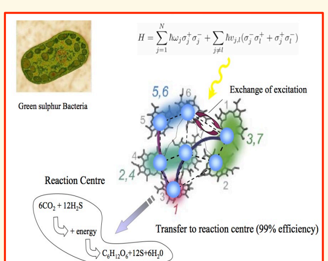
- C. Van den Broeck, J.M.R. Parrondo and R. Toral, Phys. Rev. Lett. 73, 3395 (1994)



NOISE ENHANCED STABILITY

Noise can stabilize a fluctuating or a periodically driven metastable state in such a way that the system remains in this state for a longer time than in the absence of noise.

- R.N. Mantegna and B. Spagnolo, Phys. Rev. Lett. 76, 563 (1996)

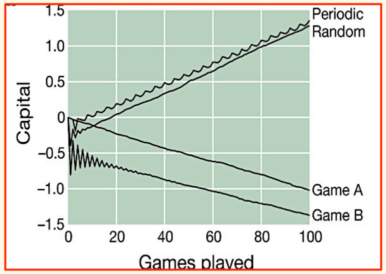


NOISE ASSISTED TRANSPORT IN BIOLOGICAL NETWORKS

Noise alters the pathways of energy transfer in biological complex networks, suppressing ineffective pathways and facilitating direct ones to the reaction centre.

- M.B. Plenio and S.F. Huelga, New J. Phys. 10, 113019 (2008)
- F. Caruso, S.F. Huelga and M.B. Plenio, Phys. Rev. Lett. 105, 190501 (2010)

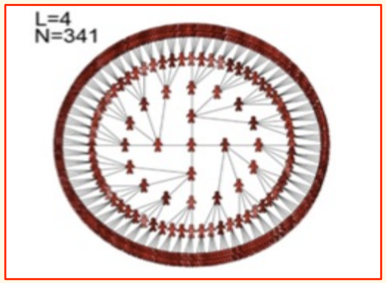
The beneficial role of noise (random strategies) in social and economic systems



MINORITY GAMES AND PARRONDO PARADOX

Optimizing agents perform worse than their non-optimized strategies, or than non-optimizing or random agents.

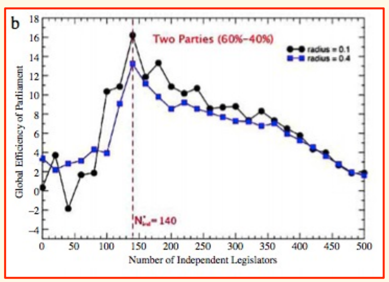
- G.P. Harmer and D. Abbott, Nature 402, 864 (1999)
- J.B. Satinover and D. Sornette, Eur. Phys. J. B 60, 369 (2007)



RANDOM STRATEGIES IN HIERARCHICAL ORGANIZATIONS

Random promotions strategies have been demonstrated to increase the efficiency of a hierarchical organization by circumventing Peter Principle's effects.

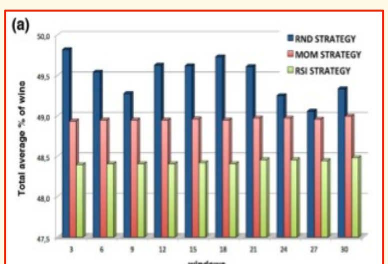
- Pluchino A., Rapisarda A. and Garofalo C., Physica A, 389, 467 (2010).
- Pluchino A., Rapisarda A. and Garofalo C., Physica A, 390 3496 (2011)



RANDOM STRATEGIES FOR SELECTING LEGISLATORS

The Parliament efficiency can be increased by the introduction of a given number of randomly selected legislators in terms of both the number of laws passed and the average social welfare obtained.

- Pluchino A., Garofalo C., Rapisarda A., Spagano S., Caserta M., Physica A 390, 3944 (2011).



RANDOM STRATEGIES IN FINANCIAL TRADING

Standard strategies, with their algorithms based on the past history of the market index, do not perform better than a purely random strategy, which, on the other hand, is also much less risky,

- Biondo A.E., Pluchino A., Rapisarda A. (2012) arXiv:1209.5881 [physics.soc-ph]

Noise and Synchronization of coupled logistic maps

Synchrony among coupled units has been extensively studied in the past decades providing important insights on the mechanisms that generate **emergent collective behaviors** in many complex systems.

- Y. Kuramoto, "*Chemical Oscillations, Waves and Turbulence*" (Springer, New York, 1984)
- A. Pikovsky, M. Rosenblum and J. Kurths, "*Synchronization. A Universal Concept in Nonlinear Sciences*", (Cambridge 2001)
- S.H. Strogatz, "*Sync: The Emerging Science of Spontaneous Order*", (Hyperion Books, 2004)

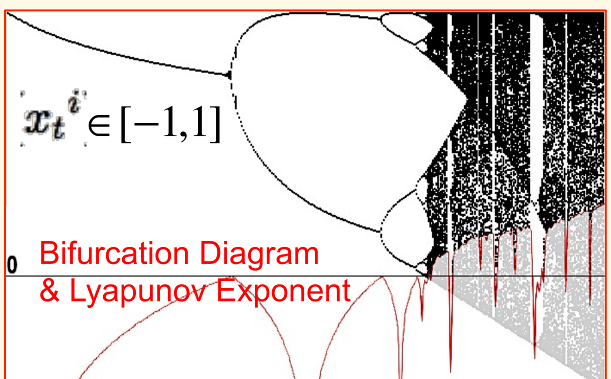
In this context **coupled maps** have often been used as a theoretical model.

- K. Kaneko , "*Simulating Physics with Coupled Map Lattices*" (World Scientific, Singapore, 1990)

KANEKO CML MODEL: A 1D LATTICE of LOCALLY COUPLED LOGISTIC MAPS

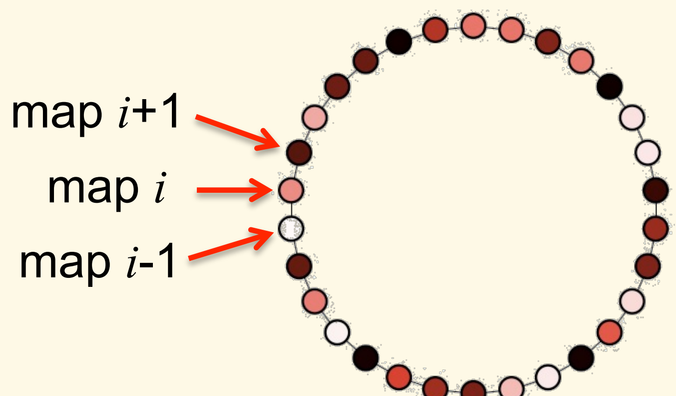
Logistic Map

$$f\left(x_t^i\right) = 1 - \mu\left(x_t^i\right)^2, \text{ with } \mu \in [0,2]$$



N Coupled Logistic Maps with periodic boundary conditions

$$x_{t+1}^i = (1-\epsilon)\, f\left(x_t{}^i\right) + \frac{\epsilon}{2}\left[f\left(x_t{}^{i-1}\right) + f\left(x_t{}^{i+1}\right)\right]$$



Strenght of the local coupling

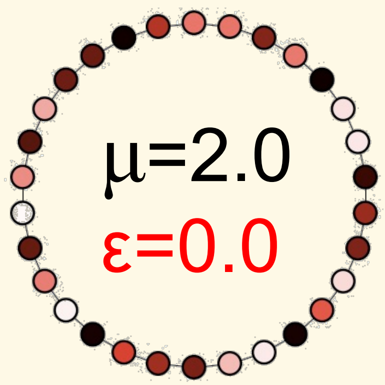
$$\epsilon \in [0,1]$$

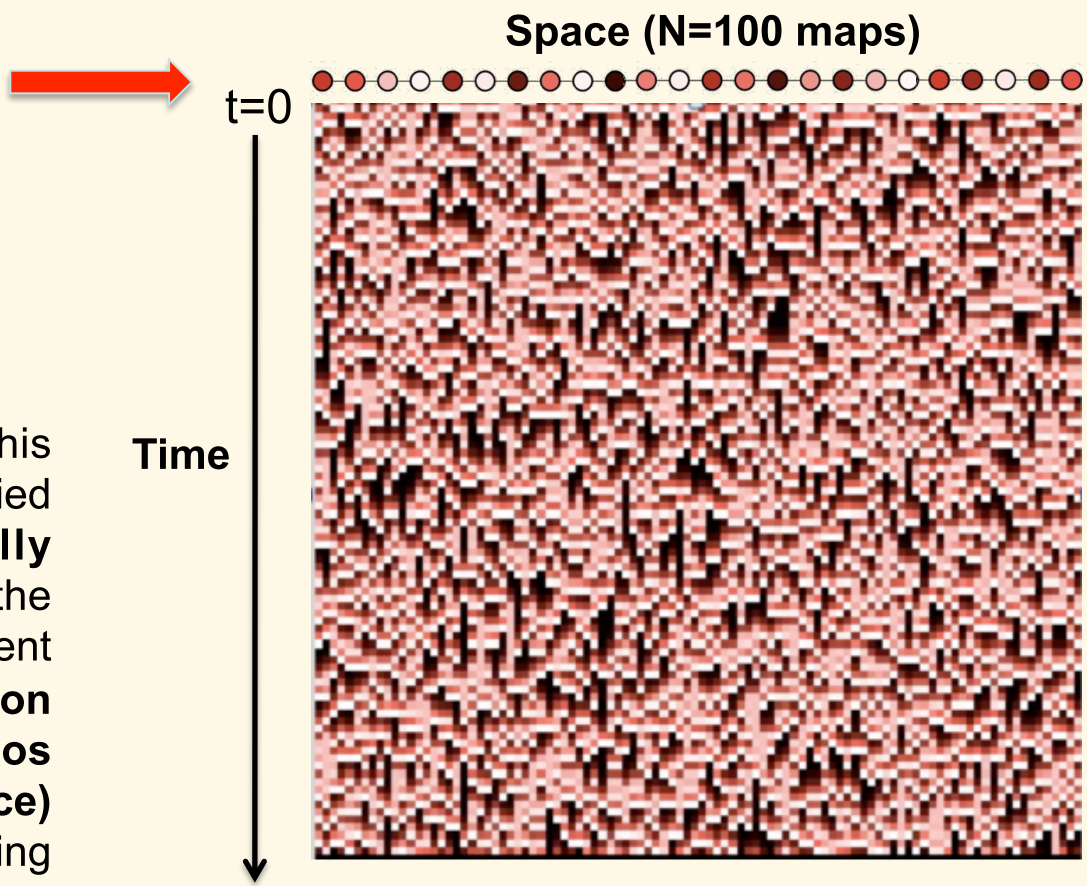
Different colors indicate different random initial conditions

Spatiotemporal chaos and synchronization patterns in the Coupled Map Lattice (CML) Model

K. Kaneko , "*Simulating Physics with Coupled Map Lattices*" (World Scientific, Singapore, 1990)

$$x_{t+1}^{i} = (1-\epsilon)\, f\left(x_t{}^{i}\right) + \frac{\epsilon}{2}\left[f\left(x_t{}^{i-1}\right) + f\left(x_t{}^{i+1}\right)\right]$$





In **absence of noise**, this model was extensively studied in particular **in the fully chaotic regime**, where the coupled maps show different **patterns of synchronization and spatiotemporal chaos (fully developed turbulence)** as function of the coupling strength ε.

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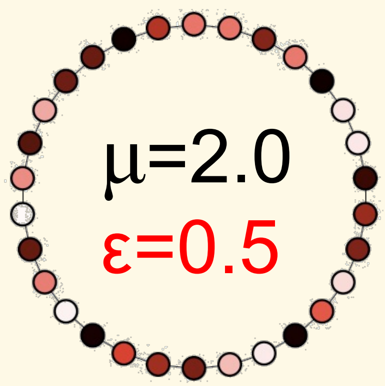

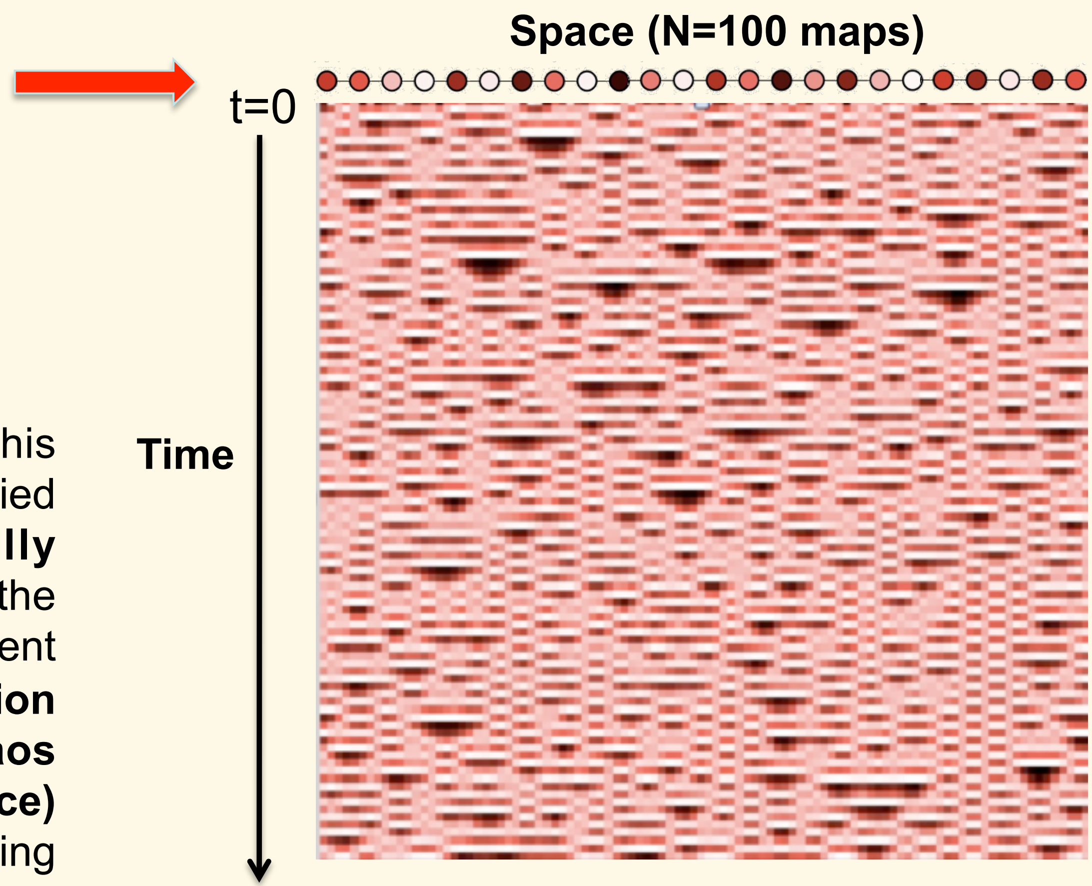

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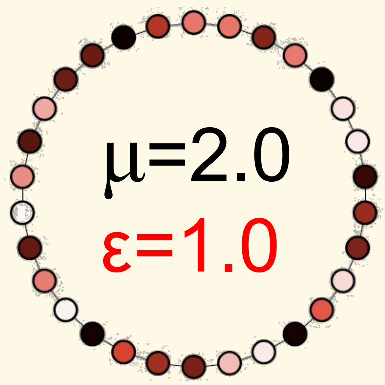
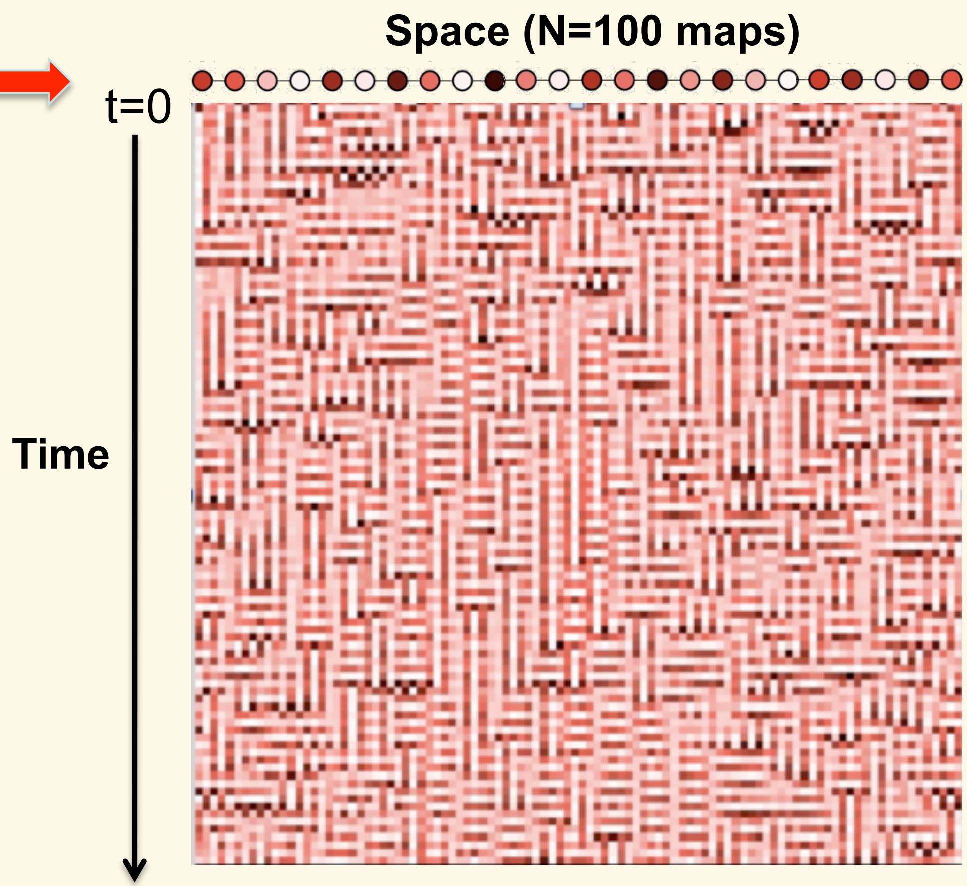




In **absence of noise**, this model was extensively studied in particular **in the fully chaotic regime**, where the coupled maps show different **patterns of synchronization and spatiotemporal chaos (fully developed turbulence)** as function of the coupling strength ε.

On-off intermittency in small-world networks of chaotic maps

C. Li and J. Fang, IEEE 0-7803-8834-8/05 (2005) 288 - 291 Vol. 1

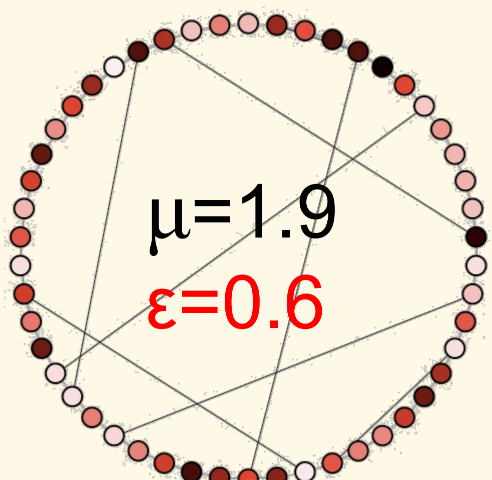
Small-world topology affects the behavior of the locally coupled logistic maps in the **fully chaotic** regime by introducing **long-range correlations** among maps. For a fixed strong coupling ε, when the **rewiring probability *p*** is slightly **less** than a critical value (0.29), the synchronous chaotic state is no longer stable and **on-off intermittency** appears.

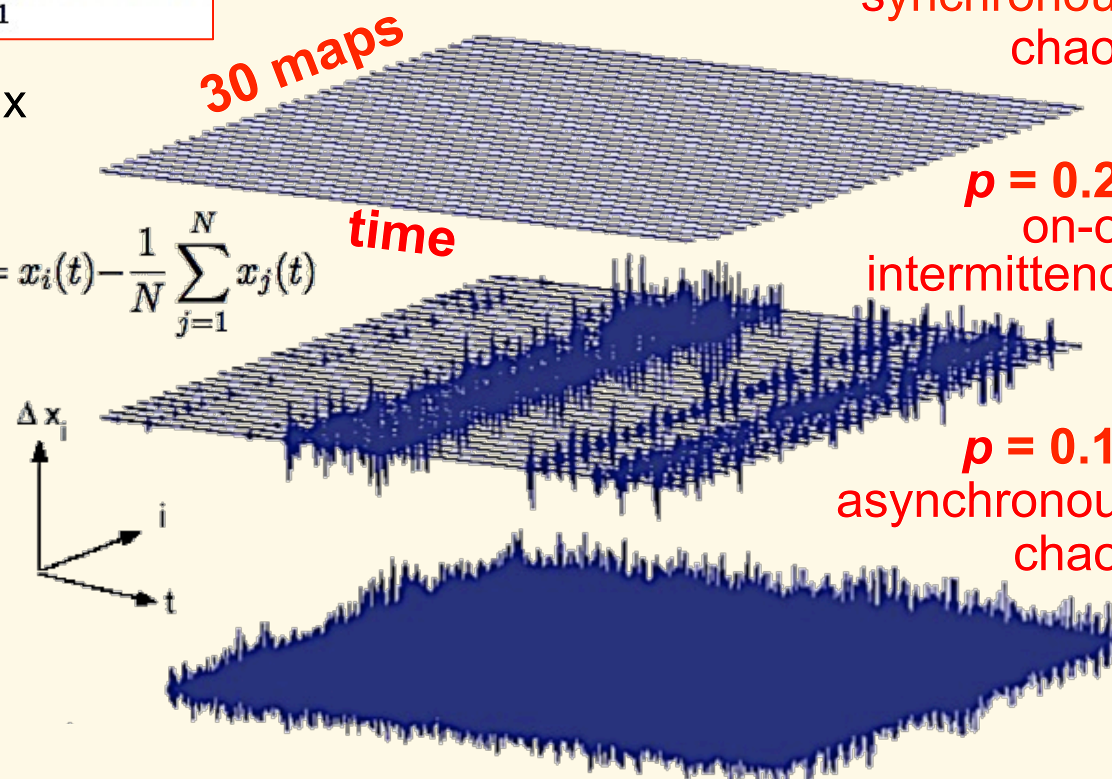
$$x_i(t+1) = (1-\epsilon) f(x_i(t)) + \frac{\epsilon}{N_i} \sum_{j=1}^{N} a_{ij} f(x_j(t))$$

$A = \{a_{ij}\}$ Connectivity Matrix

p = rewiring probability

desynchronized component: $\Delta x_i(t) = x_i(t) - \frac{1}{N} \sum_{j=1}^{N} x_j(t)$





***p* = 0.3**
synchronous chaos

***p* = 0.27**
on-off intermittency

***p* = 0.12**
asynchronous chaos

Noise induced correlations in a lattice of logistic maps at the edge of chaos

A.Pluchino, A.Rapisarda, C.Tsallis (2012) arXiv:1206.2152v1 [cond-mat]

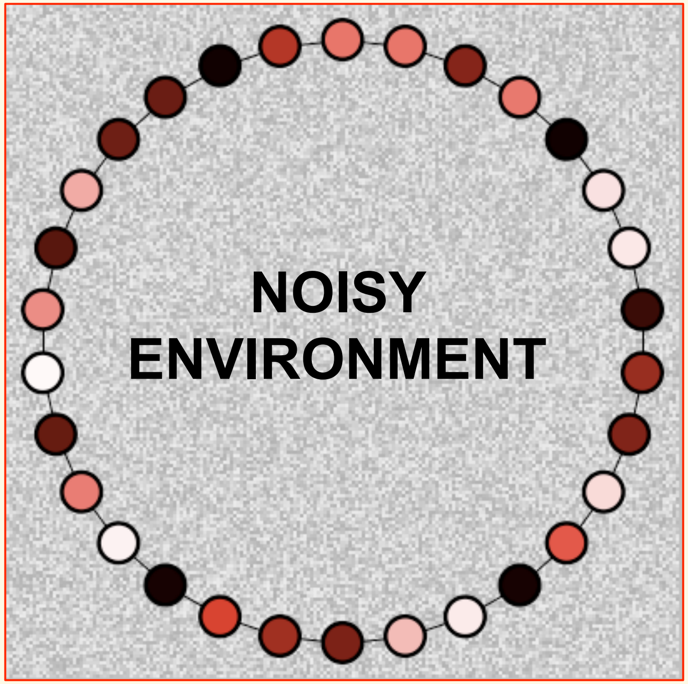
Our idea is to consider only **local coupling** and to induce **long-range correlations and intermittency** by embedding the maps in a **common noisy environment:**

$$x_{t+1}^{i} = (1-\epsilon)\, f\left({x_t}^{i}\right) + \frac{\epsilon}{2}\left[f\left({x_t}^{i-1}\right) + f\left({x_t}^{i+1}\right)\right] + \sigma(t)$$

$f\left(x_t^i\right)$ taken in module 1 with sign

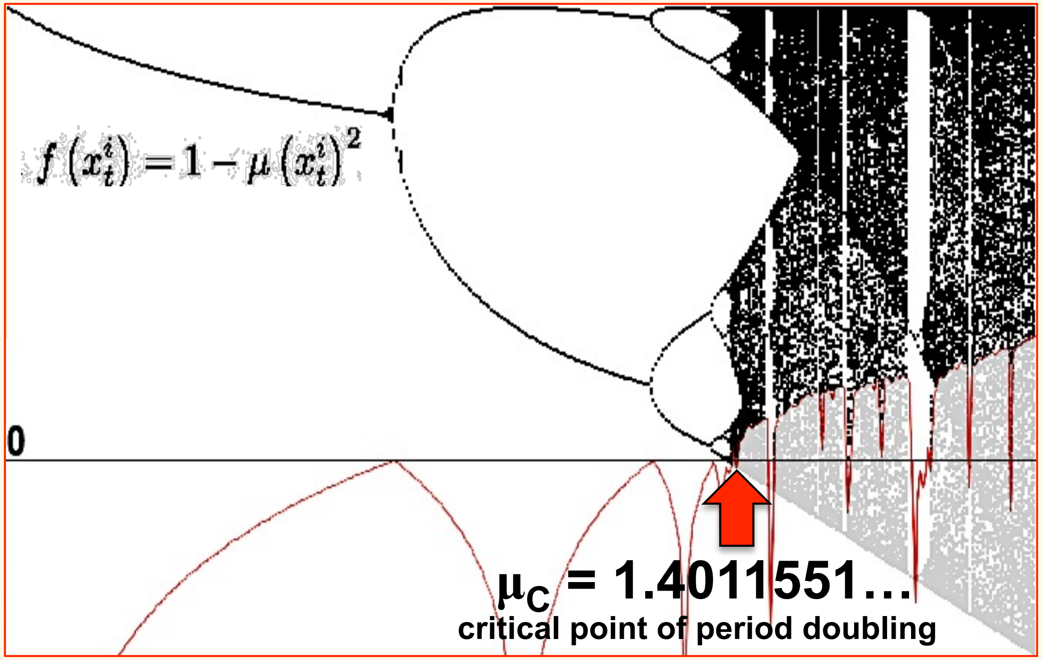
the additive noise is a random variable uniformly extracted in the interval

$\sigma(t) \in [0, \sigma_{max}]$



Noise induced correlations in a lattice of logistic maps at the edge of chaos

At variance with previous studies on coupled logistic maps we also consider them not in the chaotic regime but **at the edge of chaos**, where the *Lyapunov exponent is vanishing*:



Many biological complex systems operate frequently **at the edge of chaos and in a noisy environment**. Therefore studying the effect of a weak noise in this kind of coupled systems could be relevant in order to **understand the way in which interacting units behave in real complex systems,** like for example living cells.

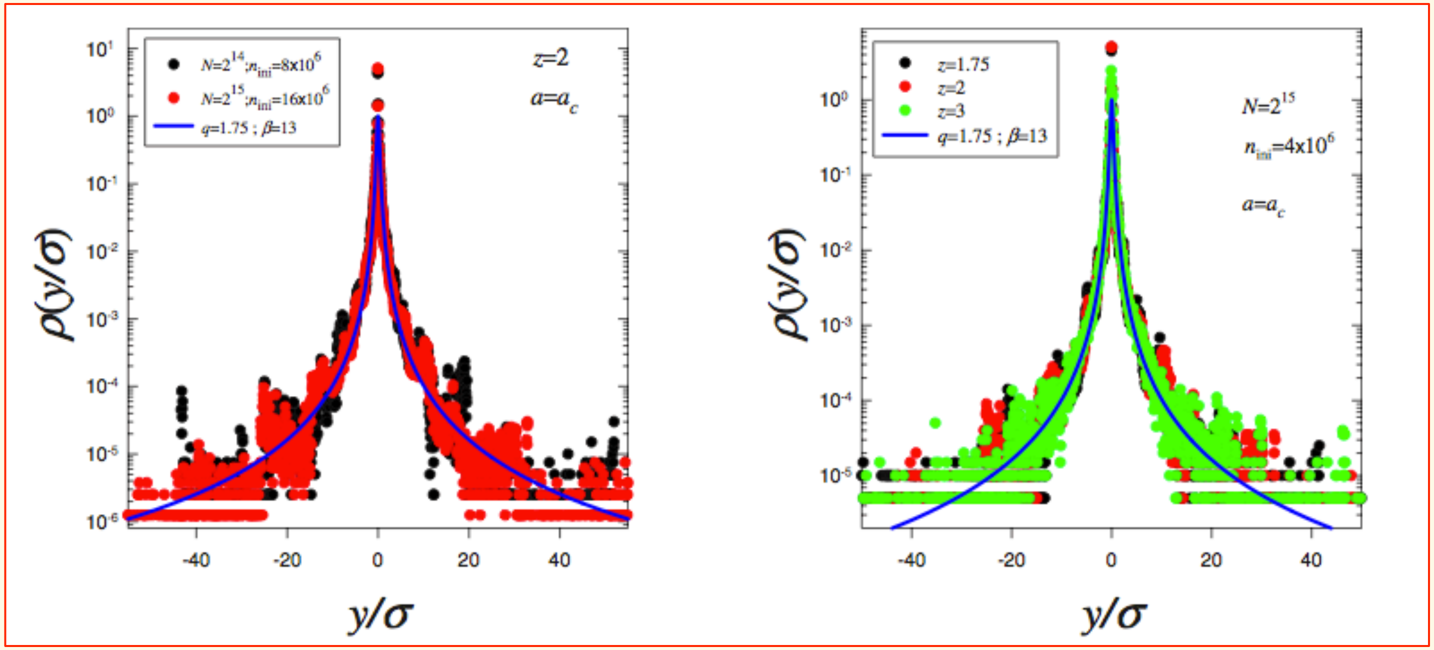
See e.g.: - D. Stokic, R. Hanel, S. Thurner, Phys. Rev. E. 77, 061917 (2008)
- R. Hanel, M. Po¨chacker, M. Scholling, S.Thurner, Plos One bf 7, e36679 (2012)

Correlations for the logistic map at the edge of chaos

The behavior of a **single logistic map at the edge of chaos** has been intensively investigated in relation to the **Central Limit Theorem** (CLT). At the **critical point** of period doubling accumulation **($\mu = \mu_c$)**, the standard CLT has been shown to be no more valid, due to **strong temporal correlations** between the iterates. In this case, the probability density converges to a ***q*-Gaussian**, in agreement with the **generalization of the CLT** in the framework of **non extensive statistical mechanics**.

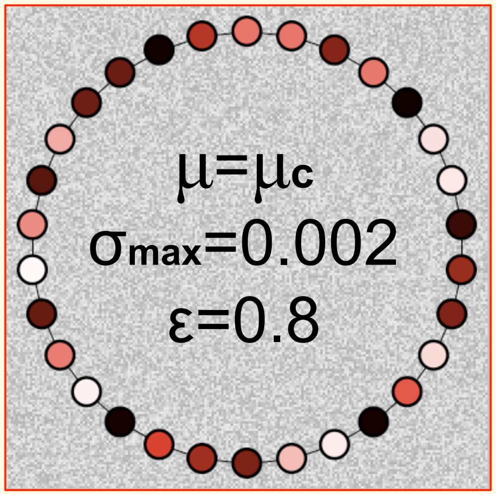
- U.Tirnakli, C.Beck and C.Tsallis, Phys. Rev. E, 75 (2007) 040106 (R)
- U.Tirnakli, C.Tsallis and C.Beck, Phys. Rev. E, 79 (2009) 056209 (R)
- S. Umarov, C. Tsallis, S. Steinberg, Milan J. math.76,307 (2008)

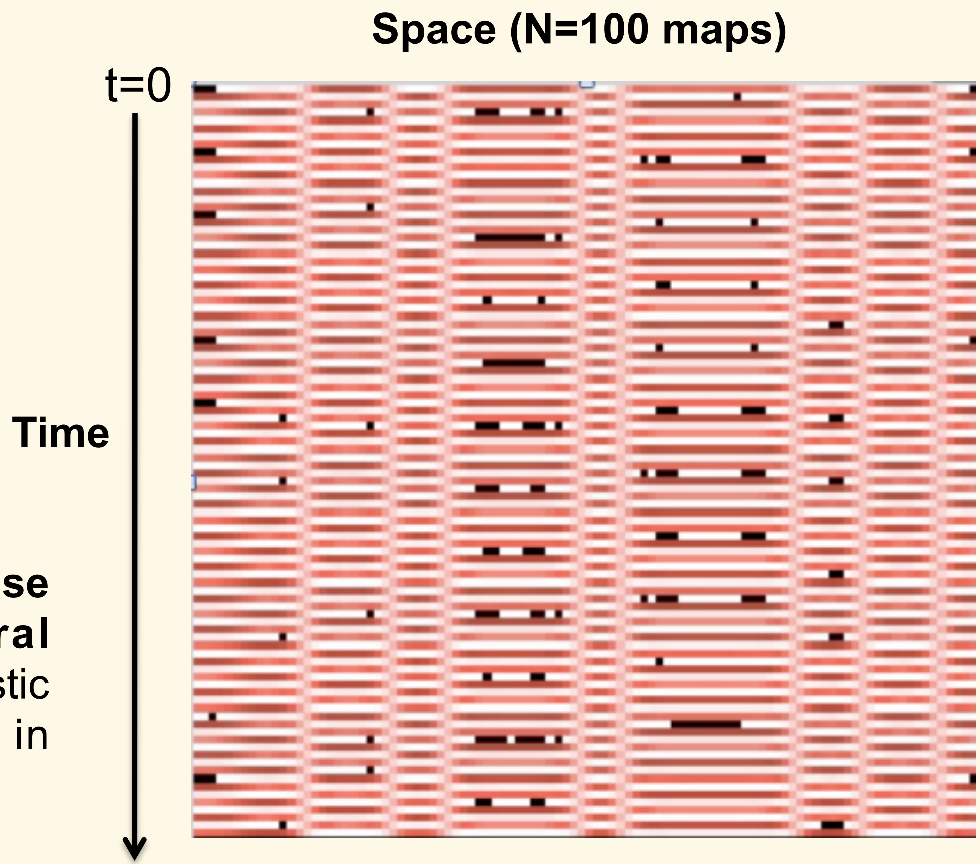
See also Andrea Rapisarda's talk



Noise induced correlations in a lattice of logistic maps at the edge of chaos

$$x^i_{t+1} = (1-\epsilon)\, f\left({x_t}^i\right) + \frac{\epsilon}{2}\left[f\left({x_t}^{i-1}\right) + f\left({x_t}^{i+1}\right)\right] + \sigma(t)$$





The addition of a **small level of noise** induces evident **spatiotemporal correlations** to the lattice of logistic maps at the edge of chaos, in presence of **strong coupling**.

Noise induced correlations in a lattice of logistic maps at the edge of chaos

In order to study these **correlations** we subtract the synchronized component and **keep the desynchronized part** of each map, considering, at every time step, the difference between the average and the single map value. Then we further consider the **average of the absolute values** of these differences over the whole system in order to measure the **distance from the synchronization regime at time t** with only one variable:

$$d_t = \frac{1}{N}\Sigma_{i=1}^{N}|x_t^i - < x_t^i > |$$

If all maps are trapped in some **synchronized pattern** then this quantity remains close to zero, otherwise **oscillations** are found.

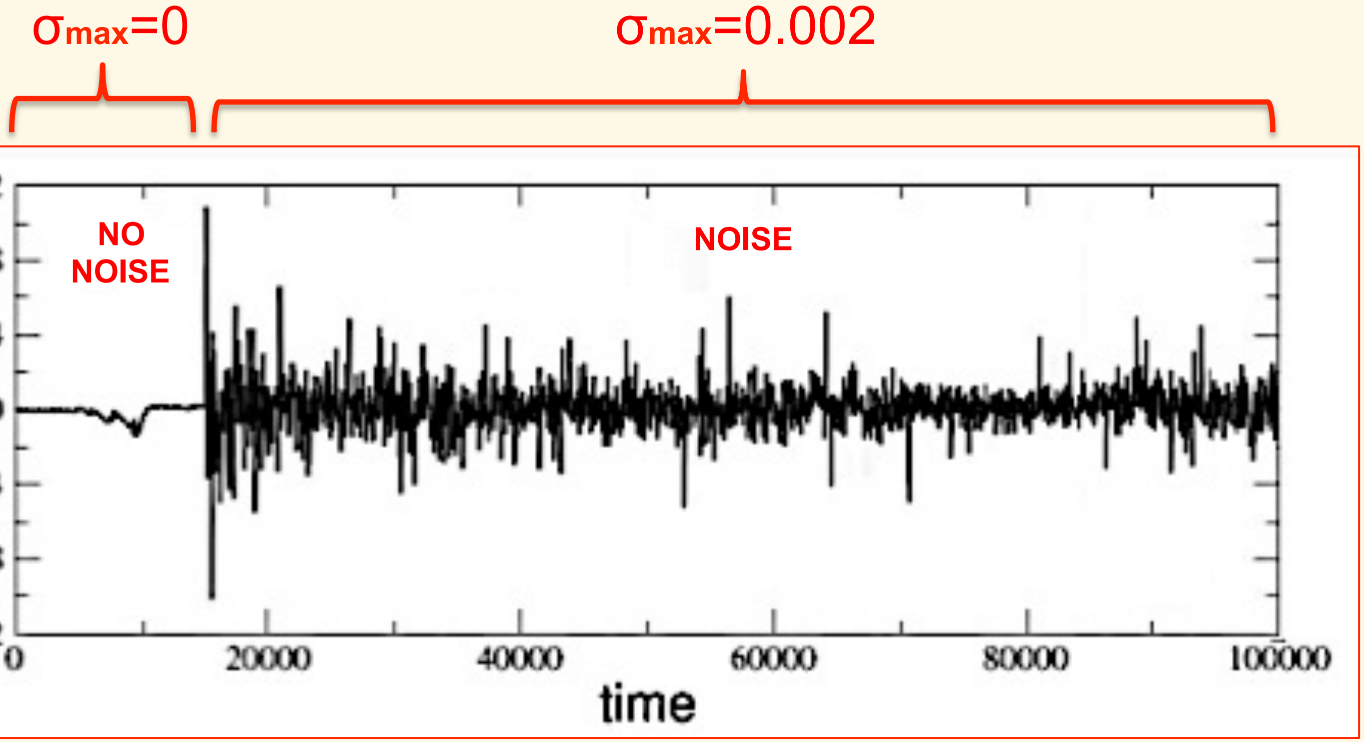
As commonly used in turbulence or in finance, we analyze these oscillations by considering the **two-time returns** Δd_t with an **interval of τ time steps**, defined as:

$$\Delta d_t = d_{t+\tau} - d_t$$

- S.Rizzo, A.Rapisarda, "*Application of superstatistics to atmospheric turbulence*" in Complexity, Metastability and Nonextensivity, World Scientific, Singapore (2005) 39
- J. Ludescher, C. Tsallis and A. Bunde, Europhys. Letters 95, 68002 (2011)

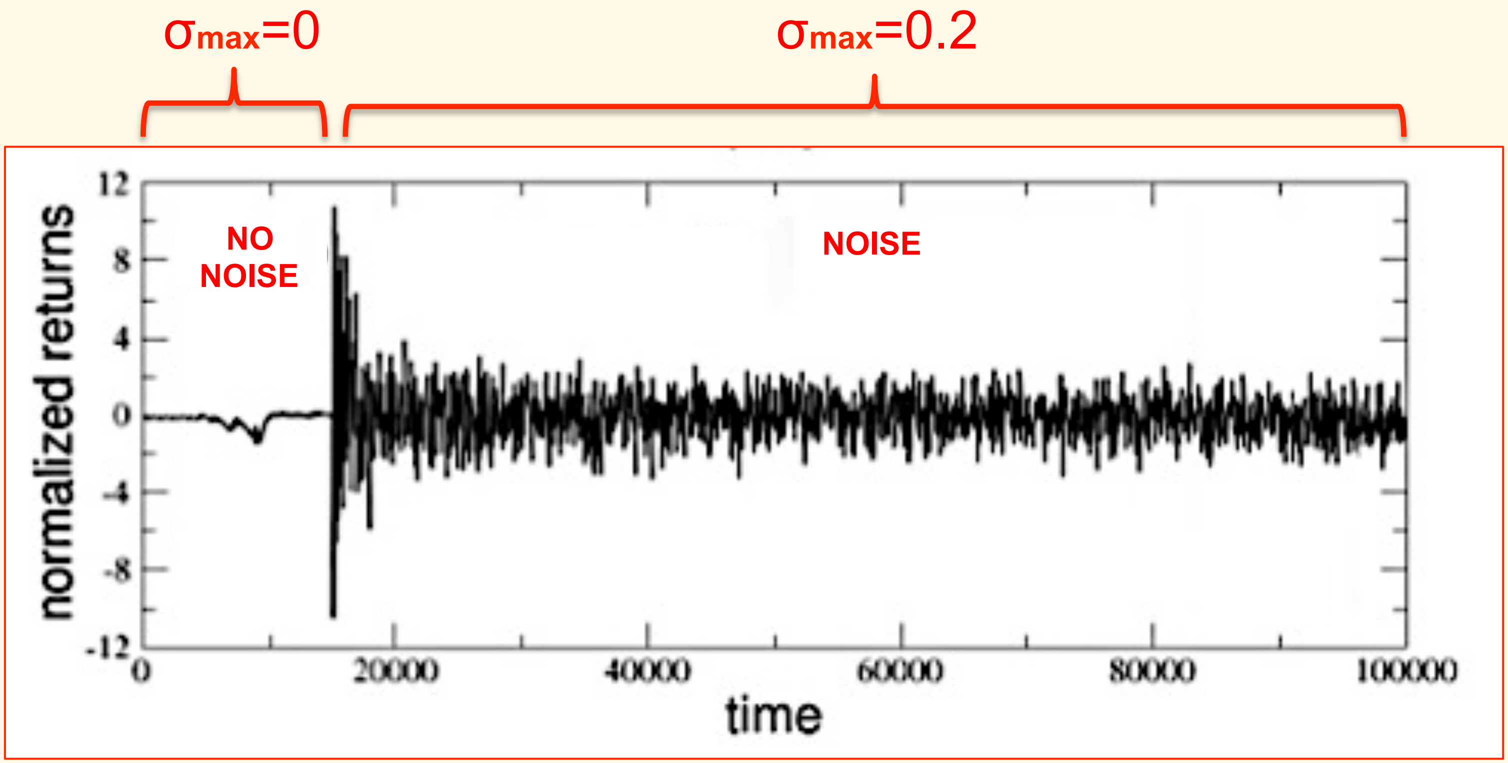
Time evolution of the two-time returns in presence of weak noise

Effect of noise in the time evolution of returns (normalized to the standard deviation of the overall sequence) for the case **N = 100, $\mu = \mu_c$ = 1.4011551..., ε = 0.8 and τ = 32 time steps.** During the first 15.000 time steps at **zero noise** (**σ_{max} = 0**) the maps remain synchronized due to the strong coupling. At time t = 15000 we **switch on the noise**, with **σ_{max} = 0.002 (weak noise)**: a clear **intermittent behavior** appears.



Time evolution of the two-time returns in presence of strong noise

The intermittent behavior **disappears** if we repeat the same simulation but with **σ_{max} = 0.2**, i.e. in presence of **strong noise.** In this case only **Gaussian fluctuations** are observed.



PDFs of normalized returns for increasing values of noise

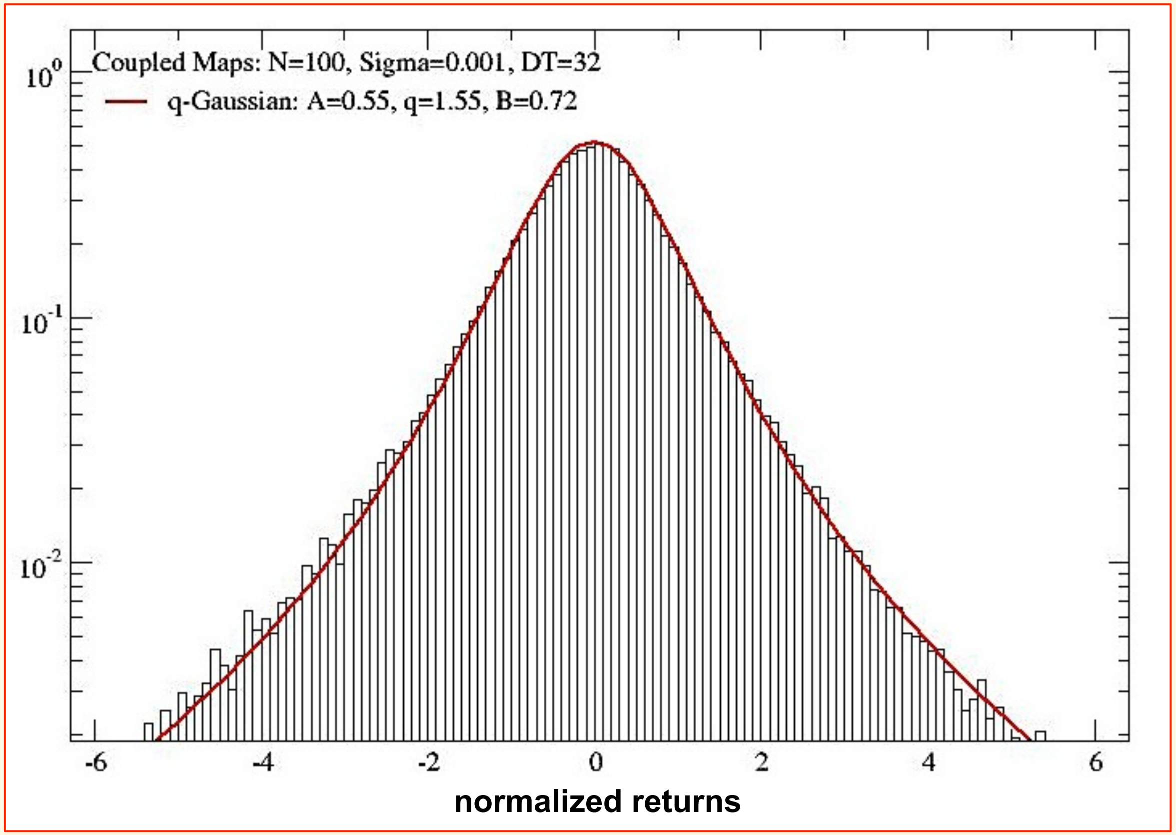
To better appreciate the transition from the intermittent to the Gaussian behavior, we plot the **probability density function (Pdf) of the normalized returns** for several **increasing values of noise.** Fat tails in the Pdfs are clearly visible only when $\sigma_{max} < 0.05$ and can be nicely reproduced by ***q*-Gaussian curves** with decreasing values of the entropic index:

$\sigma_{max}=0.001$

$q=1.55$

q-Gaussian:

$$G_q(x) = A\left[1-(1-q)\,\beta x^2\right]^{\frac{1}{1-q}}$$



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σ_{max}=0.002

q=1.54

q-Gaussian:

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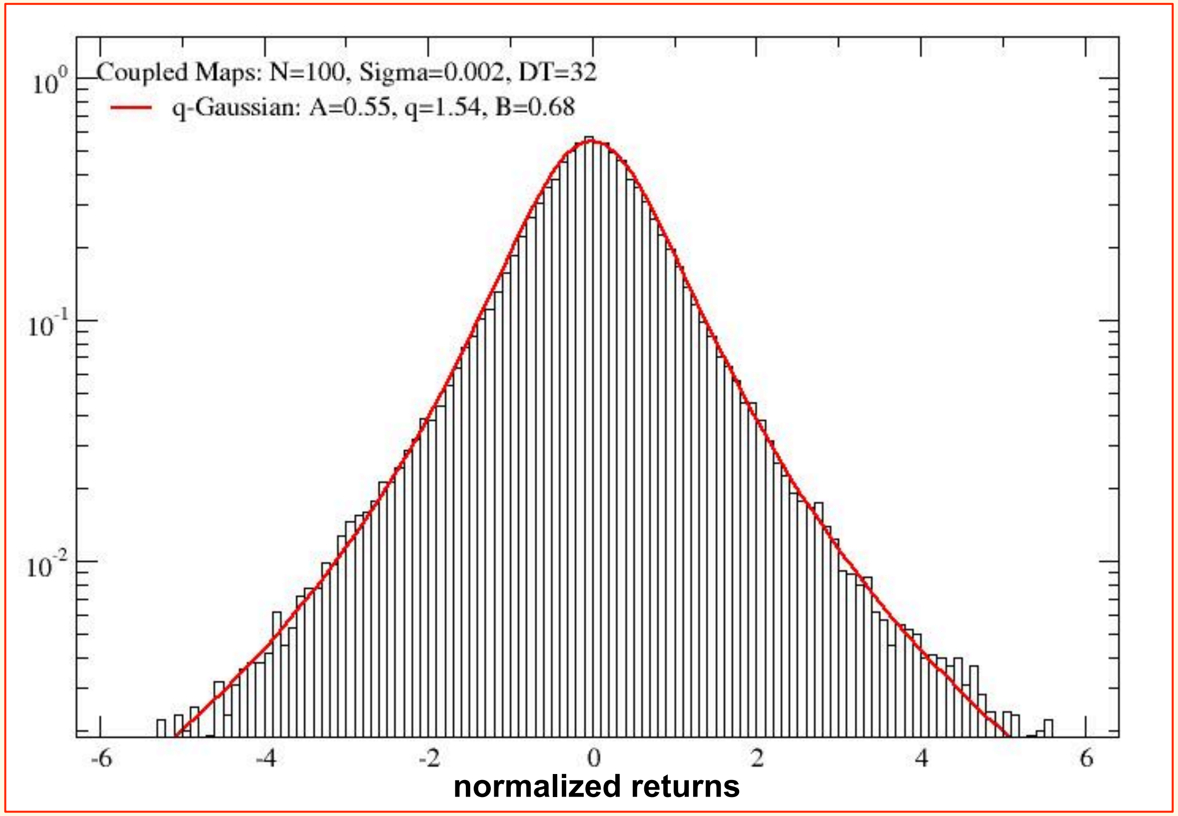

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$\sigma_{max}=0.004$

$q=1.52$

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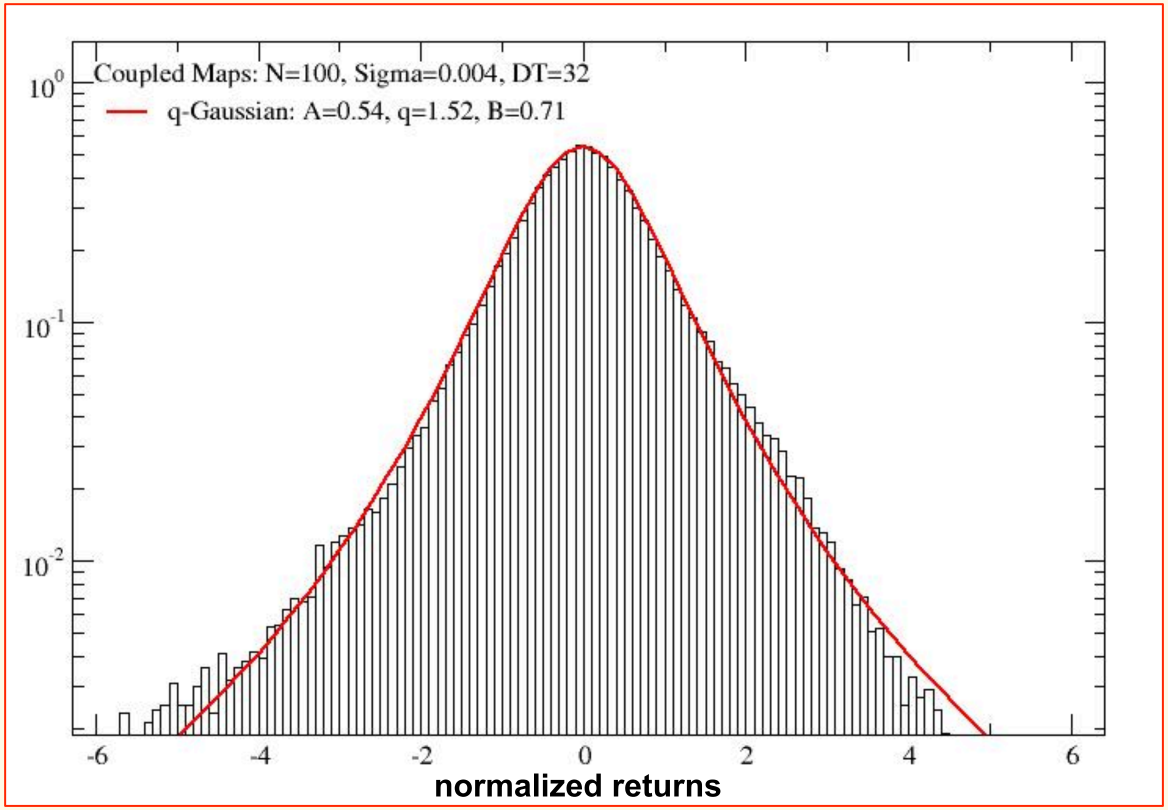

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σ_{max}=0.007

q=1.51

q-Gaussian:

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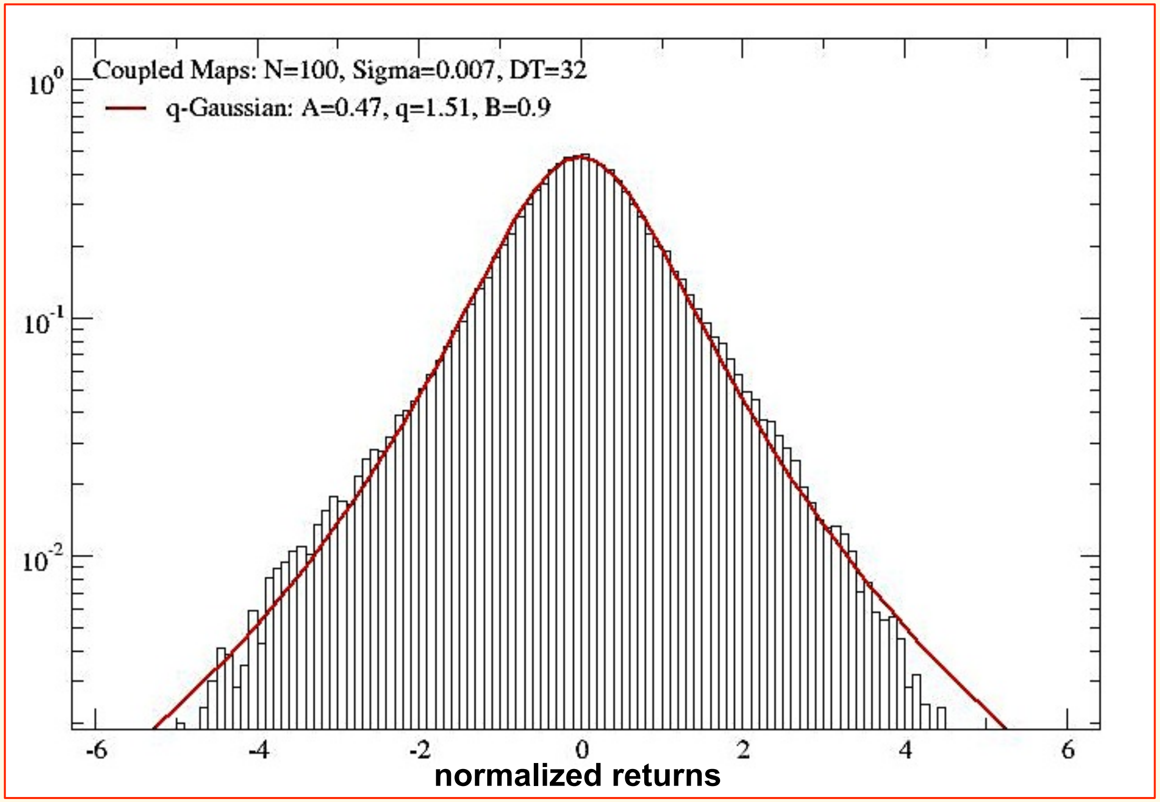

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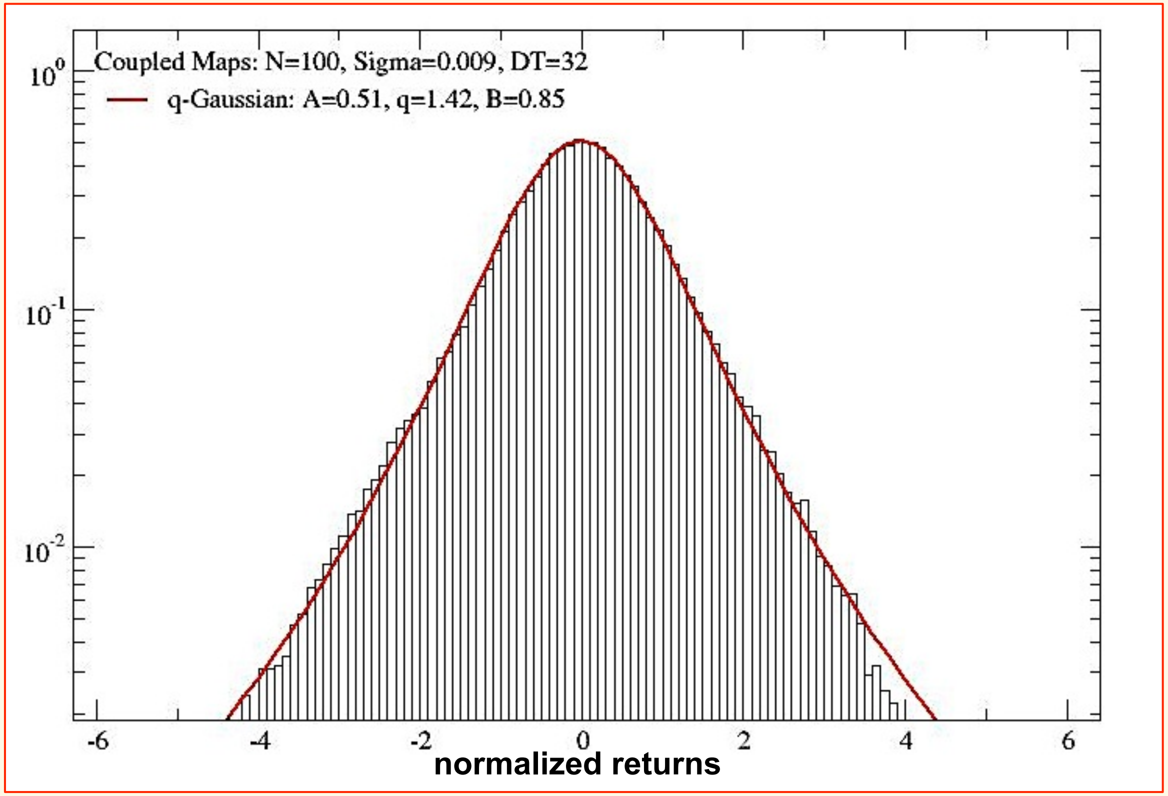
$\sigma_{max}=0.009$

$q=1.42$

q-Gaussian:

$$G_q(x) = A\left[1-(1-q)\,\beta x^2\right]^{\frac{1}{1-q}}$$



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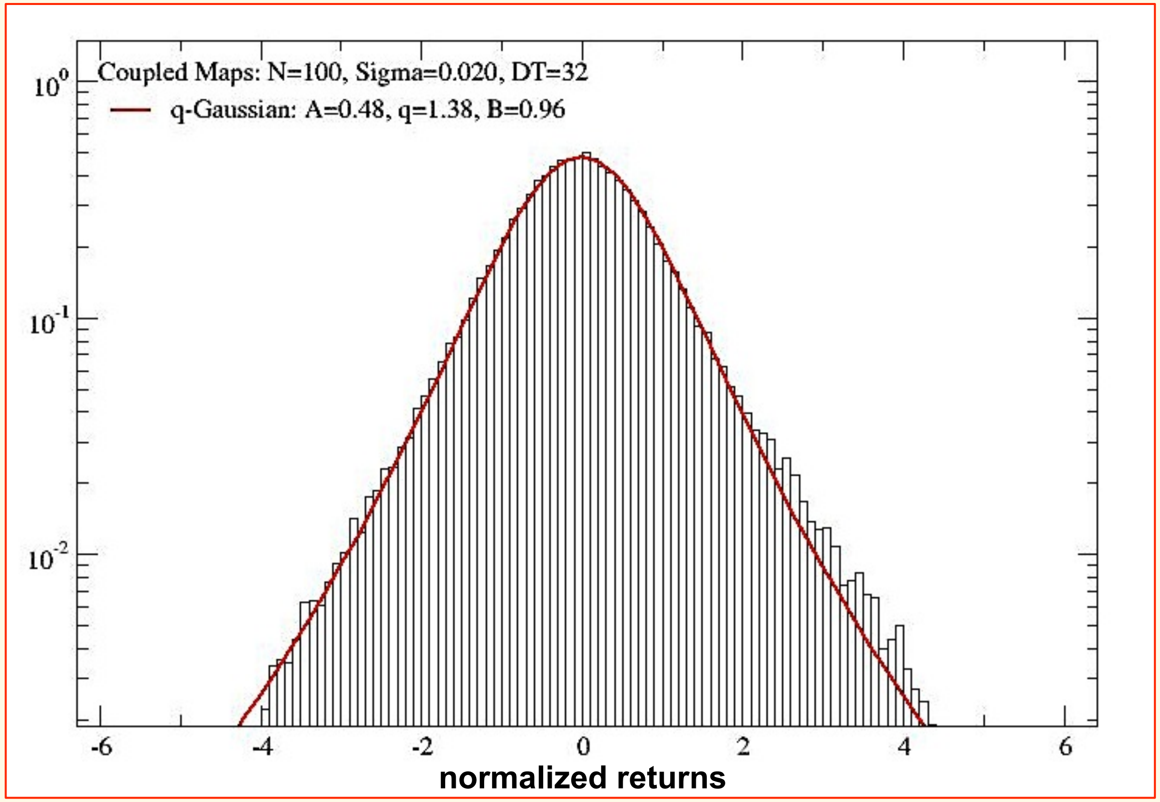
$\sigma_{max}=0.02$

$q=1.38$

q-Gaussian:

$$G_q(x) = A\left[1-(1-q)\,\beta x^2\right]^{\frac{1}{1-q}}$$



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σ_{max}=0.05

q=1.10

q-Gaussian:

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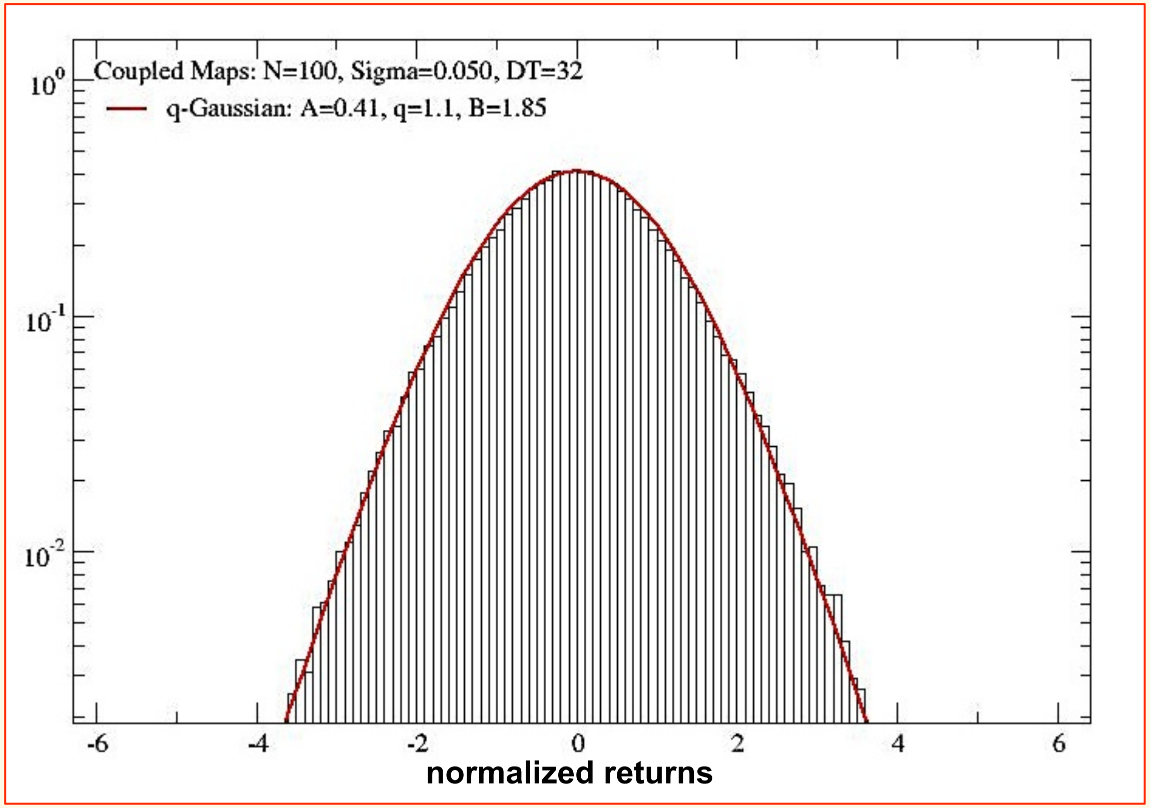

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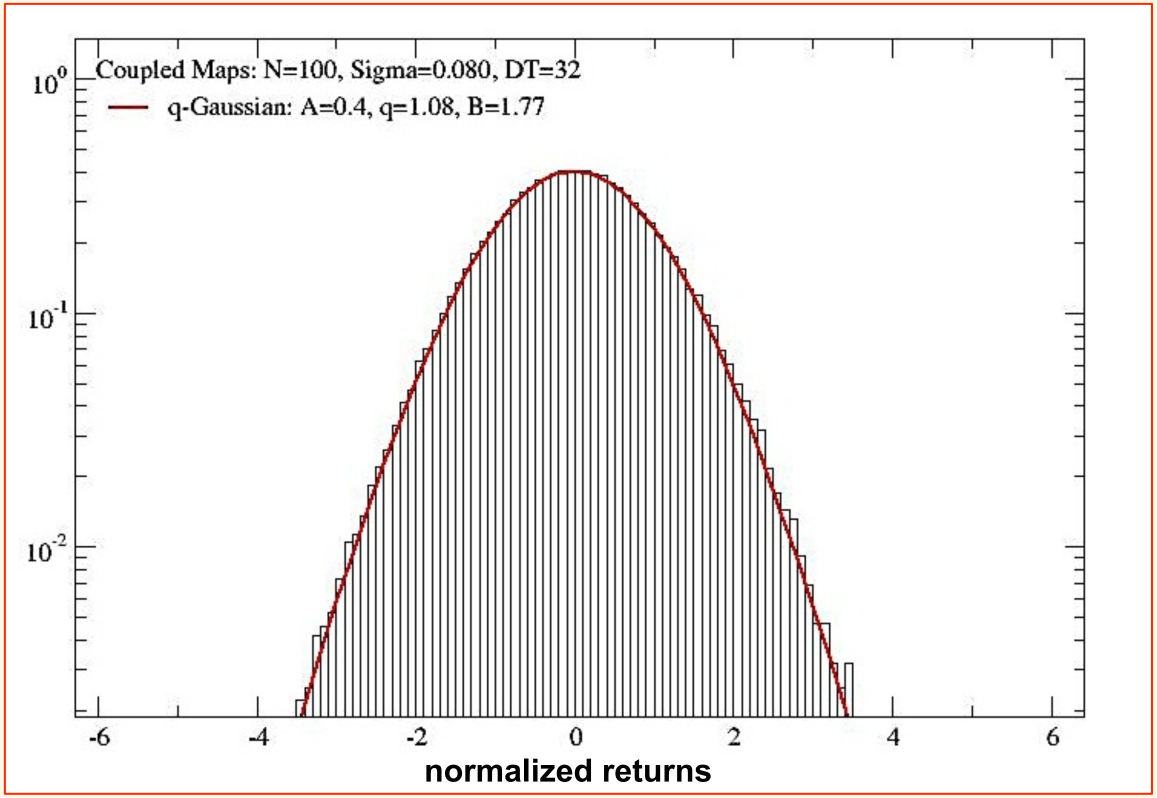
σ_{max}=0.08

q=1.08

q-Gaussian:

$$G_q(x) = A\left[1-(1-q)\,\beta x^2\right]^{\frac{1}{1-q}}$$



PDFs of normalized returns for increasing values of noise

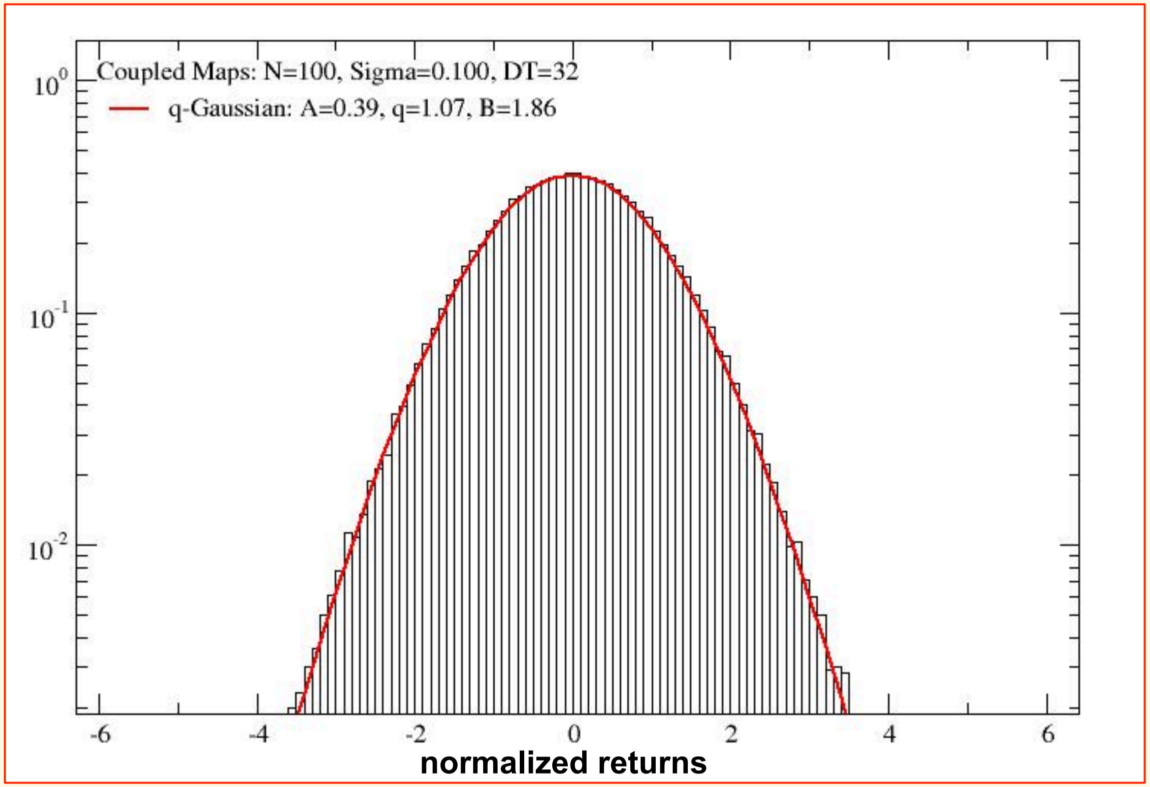
To better appreciate the transition from the intermittent to the Gaussian behavior, we plot the **probability density function (Pdf) of the normalized returns** for several **increasing values of noise.** Fat tails in the Pdfs are clearly visible only when $\sigma_{max} < 0.05$ and can be nicely reproduced by **q-Gaussian curves** with decreasing values of the entropic index:

σ_{max}=0.10

q=1.07

q-Gaussian:

$$G_q(x) = A\left[1-(1-q)\,\beta x^2\right]^{\frac{1}{1-q}}$$



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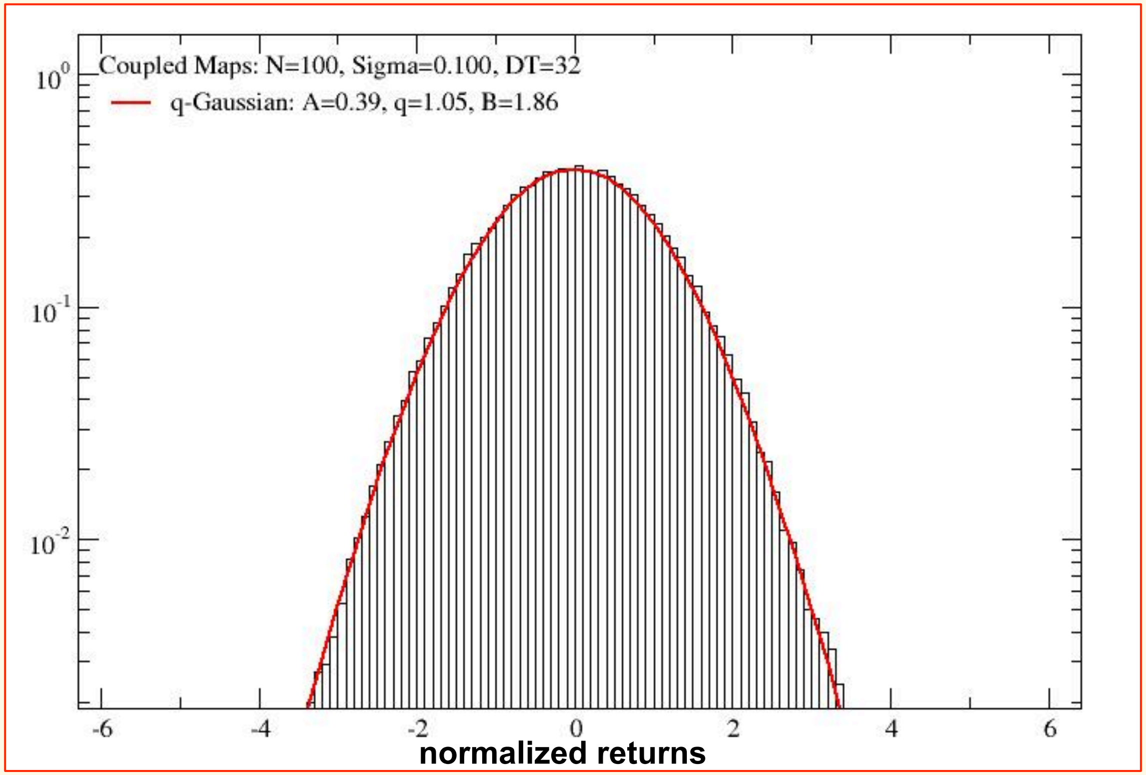
σ_{max}=0.14

q=1.05

q-Gaussian:

$$G_q(x) = A\left[1-(1-q)\,\beta x^2\right]^{\frac{1}{1-q}}$$



PDFs of normalized returns for increasing values of noise

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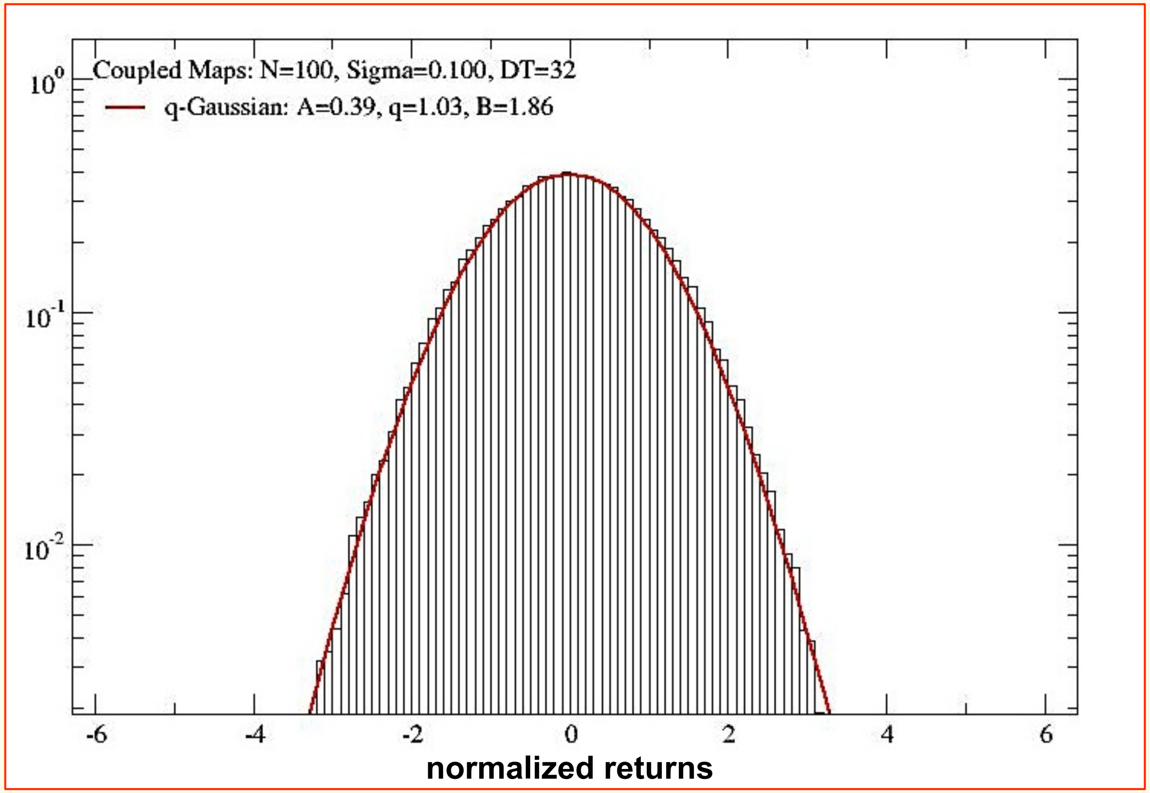
σ_{max}=0.17

q=1.03

q-Gaussian:

$$G_q(x) = A\left[1-(1-q)\,\beta x^2\right]^{\frac{1}{1-q}}$$



PDFs of normalized returns for increasing values of noise

To better appreciate the transition from the intermittent to the Gaussian behavior, we plot the **probability density function (Pdf) of the normalized returns** for several **increasing values of noise.** Fat tails in the Pdfs are clearly visible only when $\sigma_{max} < 0.05$ and can be nicely reproduced by **q-Gaussian curves** with decreasing values of the entropic index:

$\sigma_{max}=0.20$

$q=1.015$

q-Gaussian:

$$G_q(x) = A\left[1-(1-q)\,\beta x^2\right]^{\frac{1}{1-q}}$$

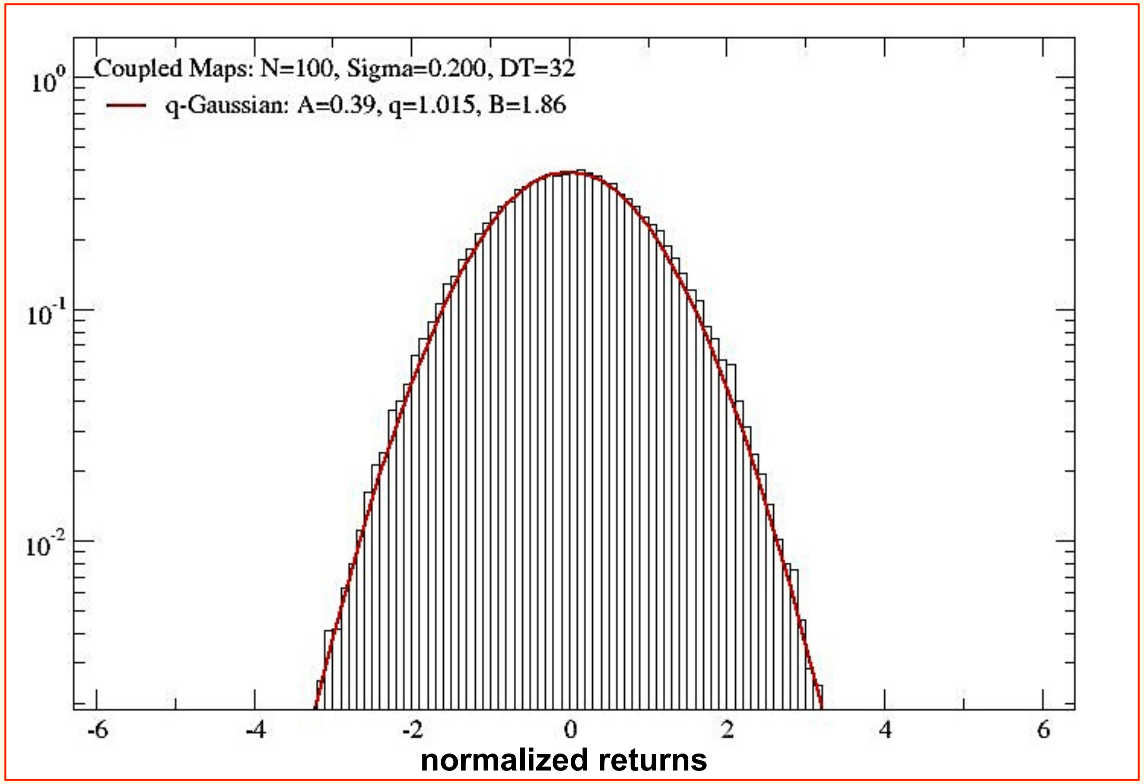
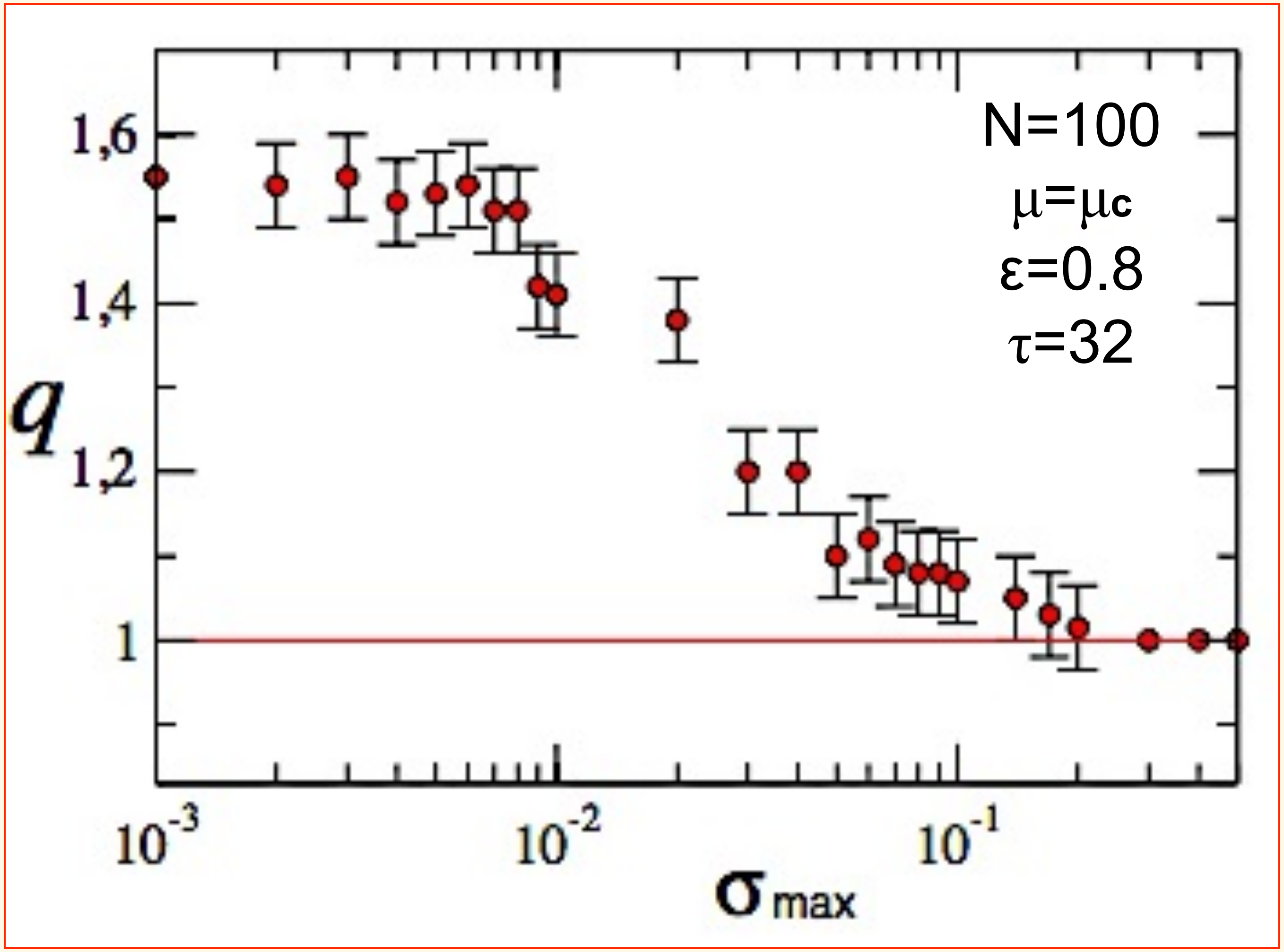


Diagram of *q* versus σ



Test of *q*-logarithm

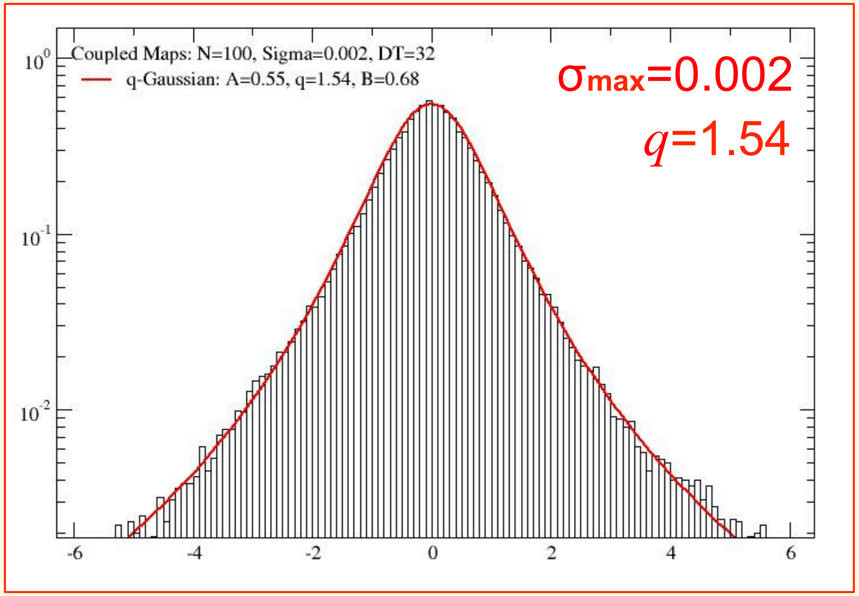
As a test to verify the accuracy of the q-Gaussian fit, we plot the **q-logarithm** of the pdf for the **case σ_{max}=0.002**, normalized to its peak, as function of x^2, and we verify that a q-logarithm curve with q = 1.54 **fits very well** the simulation points with a **correlation coefficient equal to 0.9958**.

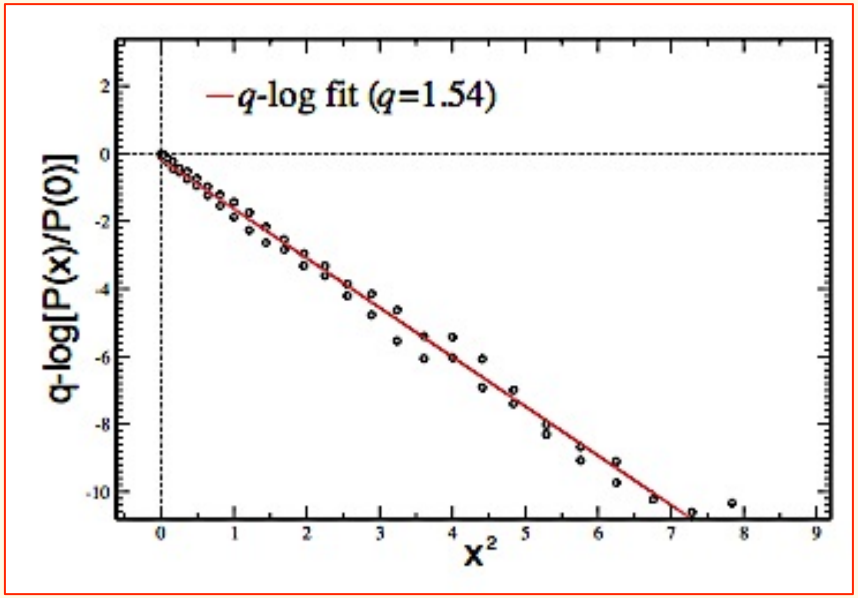
q-Gaussian:

$$G_q(x) = A\left[1-(1-q)\,\beta x^2\right]^{\frac{1}{1-q}}$$

q-logarithm:

$$\ln_q z \equiv [z^{1-q}-1]/[1-q]$$

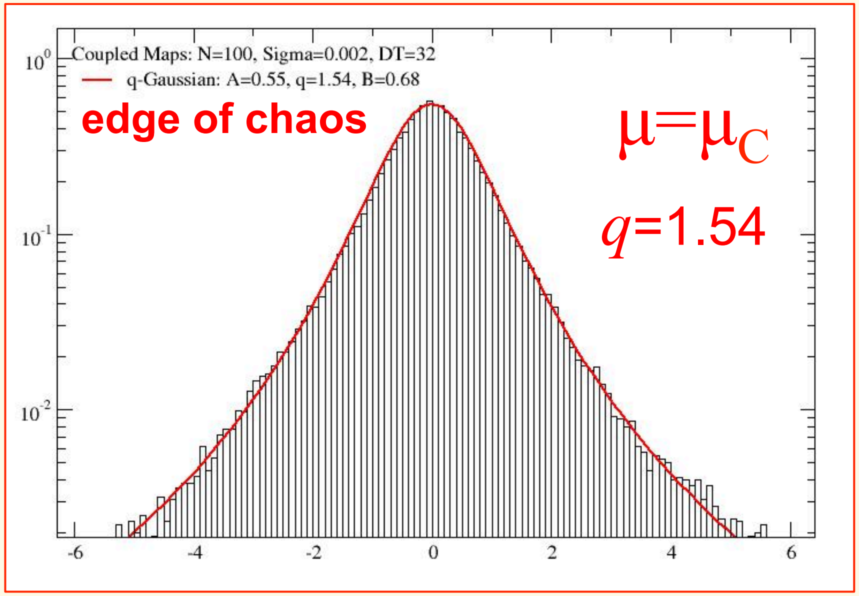


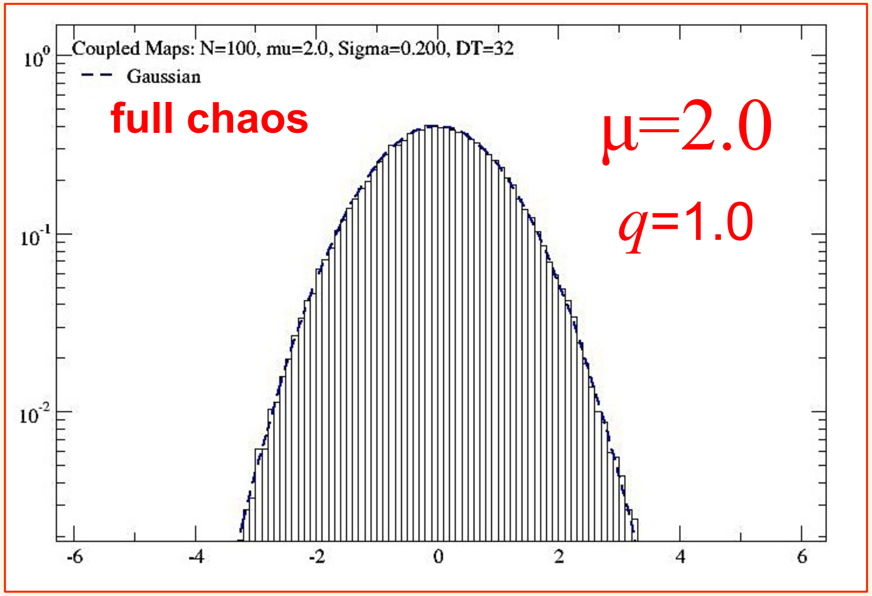


Pdf of normalized returns in the fully chaotic regime

Finally, we show that **the edge of chaos condition is strictly necessary** for the emergence of intermittency and strong correlations in presence of a small level of noise. In fact, if we consider the maps in the **fully chaotic regime**, i.e. with $\mu = 2$ instead of $\mu = \mu_c$, and leaving all the other parameters unchanged, we obtain a **Gaussian Pdf** of returns.

N=100, σ_{max}=0.002, ε=0.8, τ=32

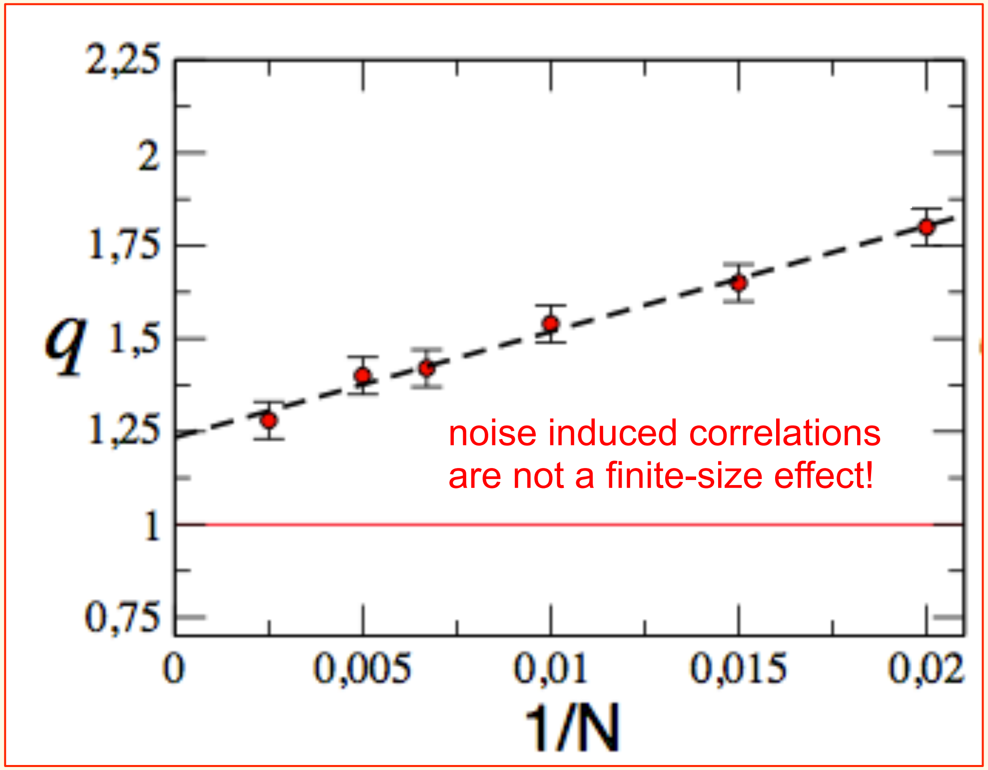




Entropic index q as function of the other parameters

Considering the value of the entropic index q as a **measure of the correlations** induced by the noisy environment on our system of coupled maps at the edge of chaos, it is worthwhile to explore how this value **changes** as function, not only of the noise σ_{max} , but also of the **number of maps**, the **coupling strength** and the **returns time interval**.

q versus the number of maps N



σ_{max}=0.002

$\mu=\mu_c$

ε=0.8

τ=32

Entropic index q as function of the other parameters

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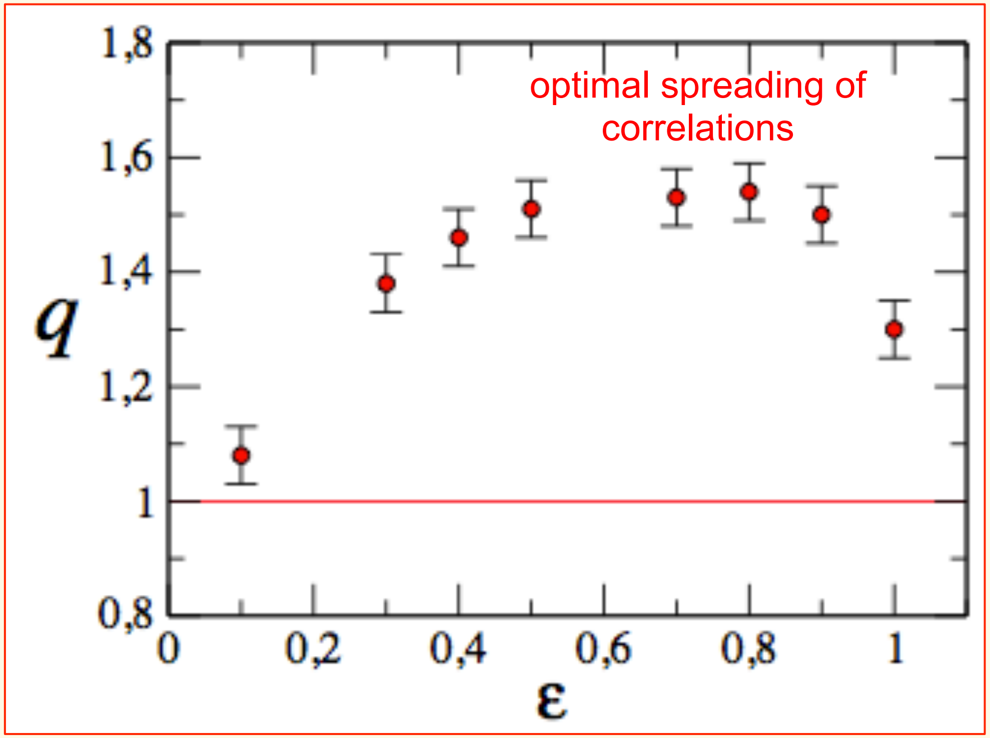
q versus the coupling strength ε



N=100

$\mu=\mu_c$

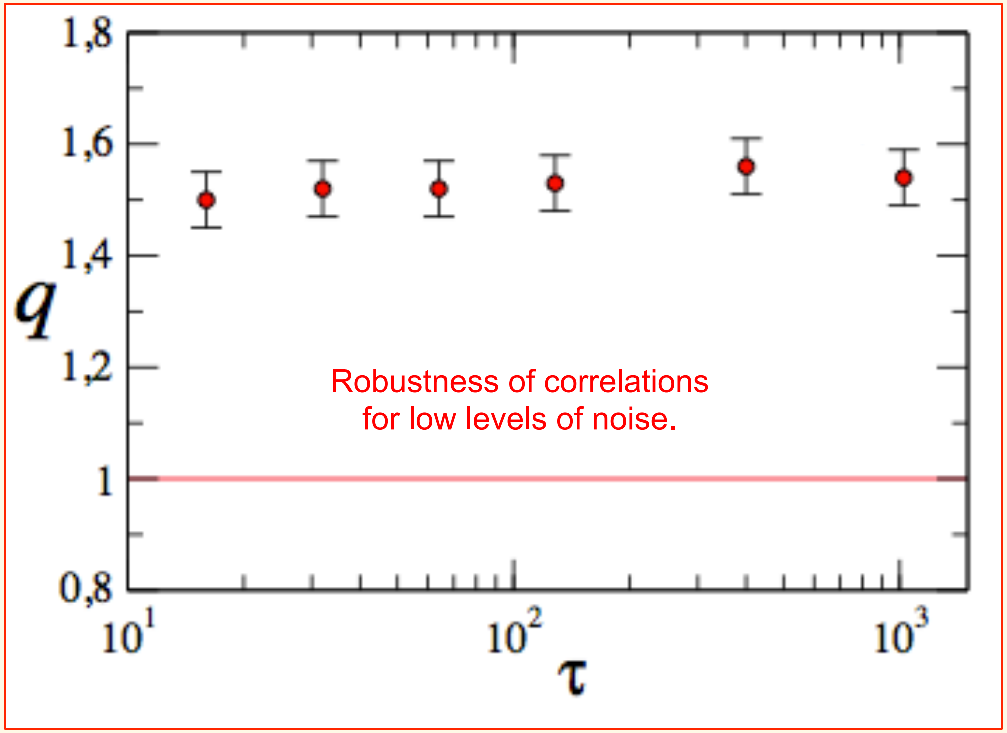
σ_{max}=0.002

τ=32

Entropic index q as function of the other parameters

Considering the value of the entropic index q as a **measure of the correlations** induced by the noisy environment on our system of coupled maps at the edge of chaos, it is worthwhile to explore how this value **changes** as function, not only of the noise σ_{max} , but also of the **number of maps**, the **coupling strength** and the **returns time interval**.

q versus the returns time interval τ



N=100

$\mu=\mu_c$

$\varepsilon=0.8$

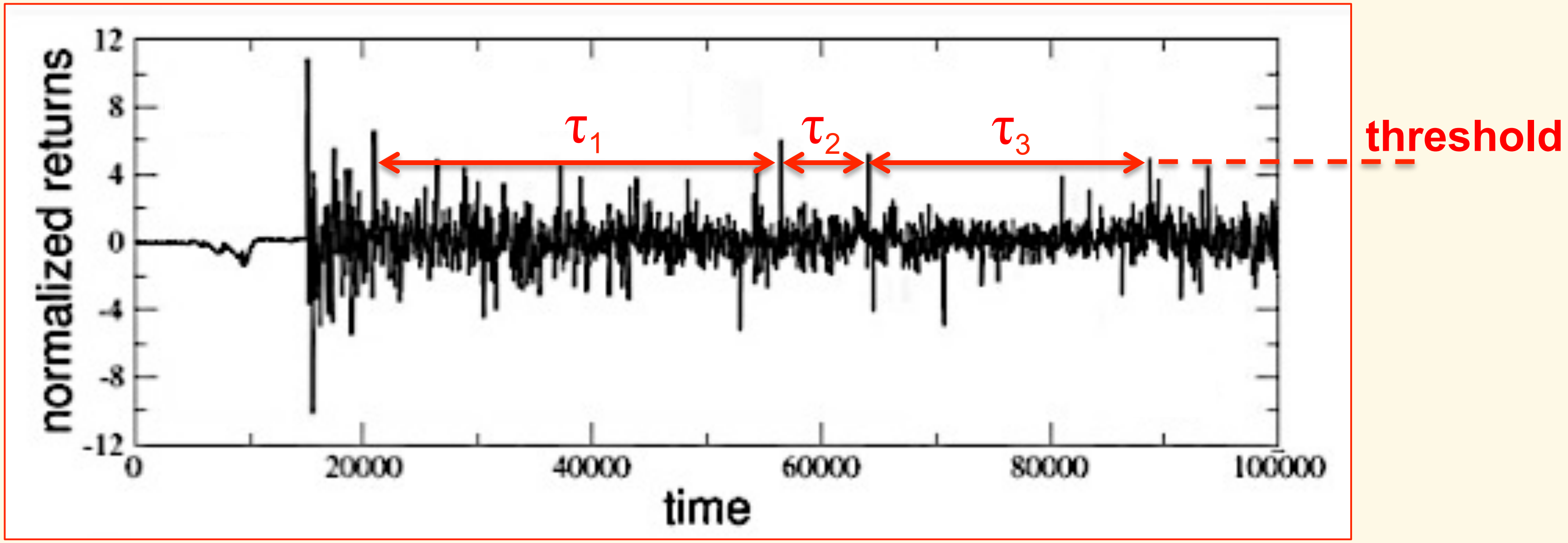
$\sigma_{max}=0.002$

Analysis of the interoccurence times

Long-term correlations in a system typically yield powerlaw asymptotic behaviors in various physically relevant properties. In studies of **financial markets***, it was recently observed **power-law decays** in the so-called '**interoccurrence times**' between sub sequential peaks in the fluctuating time series of returns. If we fix a given **threshold**, the sequence of the interoccurrence times (τ_i) results to be well defined and it is then possible to study its Pdf for our system of coupled maps at the edge of chaos.

* M.I. Bogachev and A. Bunde, Phys. Rev. E 78, 036114 (2008)

N=100, σ_{max}=0.002, $\mu=\mu_c$, ε=0.8, τ=32



Analysis of the interoccurence times in financial markets

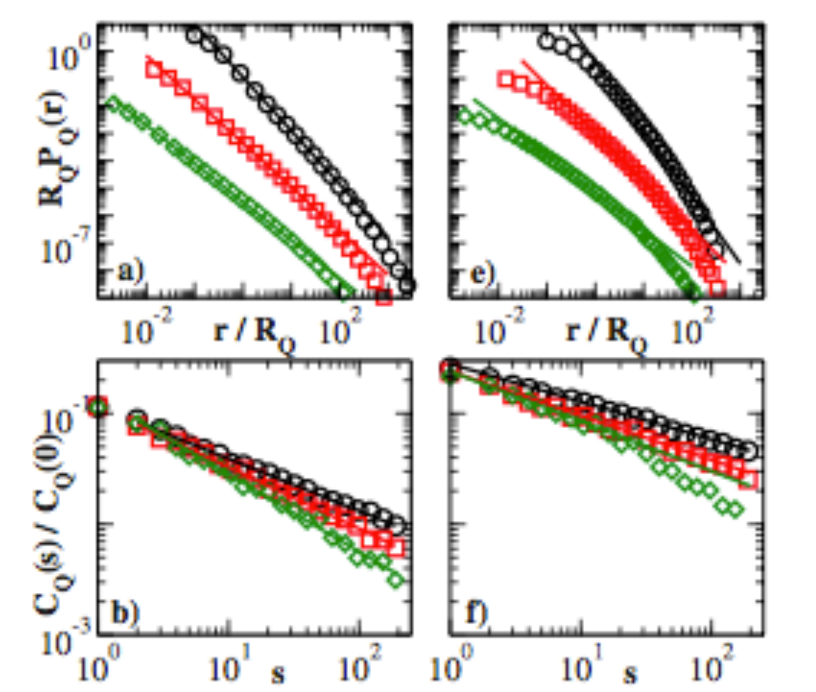
PHYSICAL REVIEW E **78**, 036114 (2008)

Memory effects in the statistics of interoccurrence times between large returns in financial records

Mikhail I. Bogachev and Armin Bunde
Institut für Theoretische Physik III, Justus-Liebig-Universität Giessen, 35392 Giessen, Germany
(Received 11 February 2008; revised manuscript received 18 June 2008; published 22 September 2008)

Pdfs of return intervals:

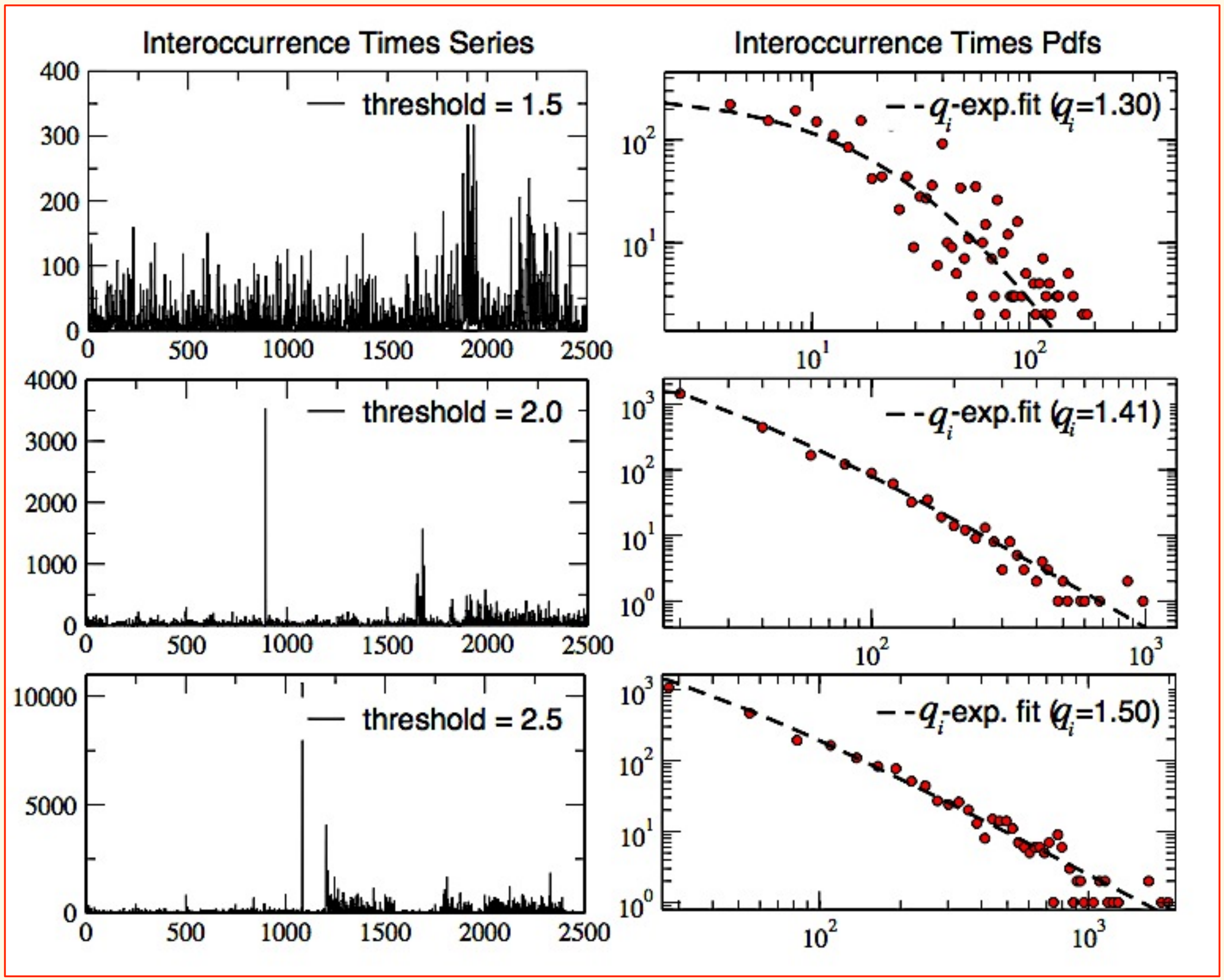
ACFs of return intervals:



Analysis of the interoccurence times

In complete analogy with what was observed for financial data, we found a **power-law behavior** for the interoccurence times pdfs that can be satisfactorily fitted with ***q*-exponential** curves y≈ $[1-(1-q_i)\tau_i/\tau_{q_i}]^{1/1-q_i}$, whose values of q_i strictly depend on the threshold.

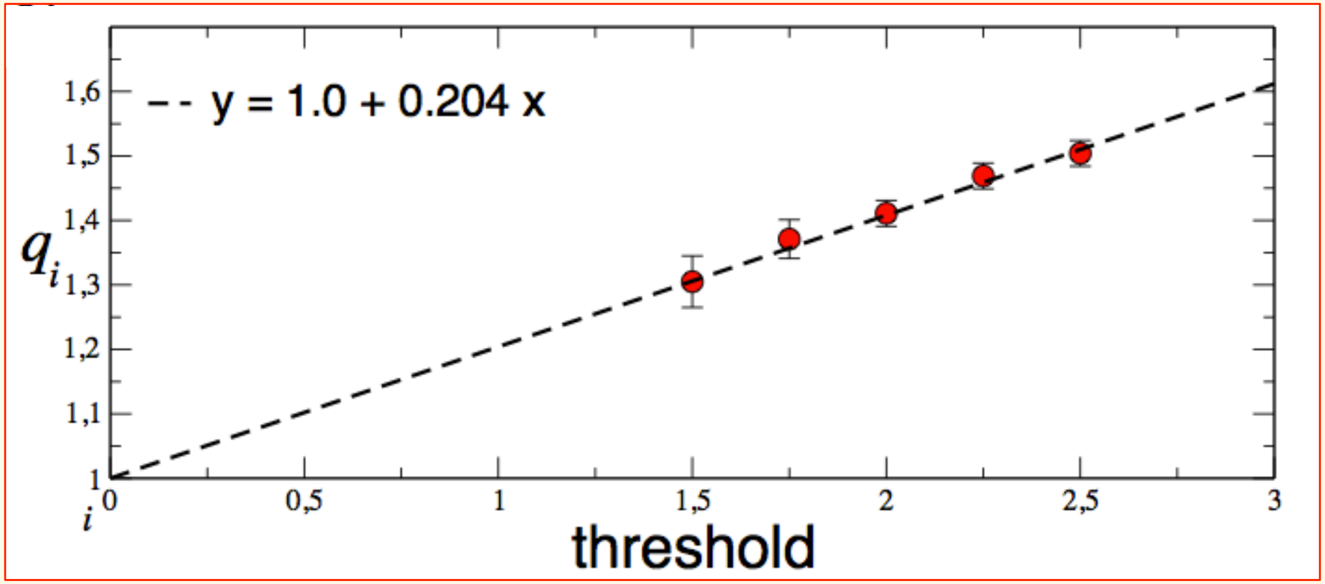
N=100, σ_{max}=0.002, μ=μc, ε=0.8, τ=32



Analysis of the interoccurence times

This can be considered as a **further footprint of the complex emergent behavior** induced on the system by the small level of noise considered. Interestingly enough, in the limit of **vanishing threshold**, q_i approaches unity, i.e., the **behavior becomes exponential**, which is precisely what was systematically observed in financial data*.

*J. Ludescher, C. Tsallis and A. Bunde, Europhys. Letters 95, 68002 (2011)

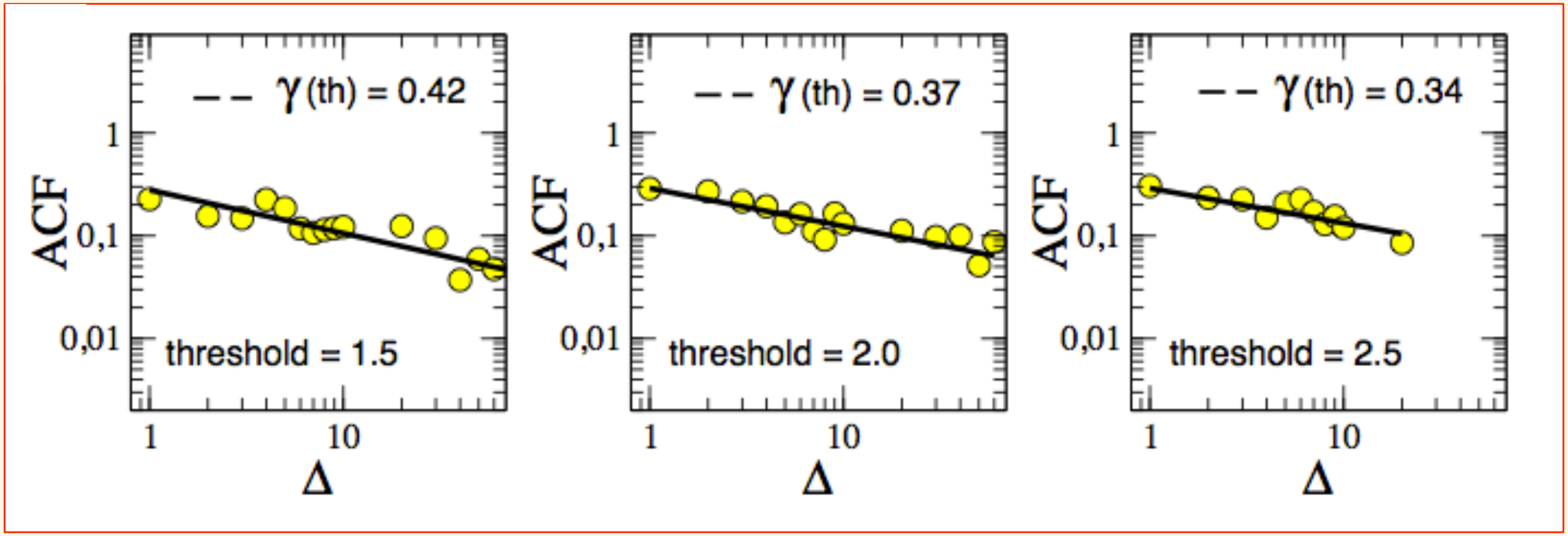


Analysis of the interoccurence times

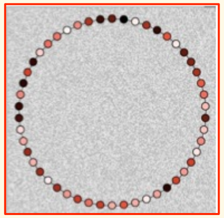
Finally, we also calculated the **auto-correlation function** (ACF)

$$C_{th}(\Delta) = A' \sum_{k}^{L-\Delta} (\tau_i(k) - <\tau_i>)(\tau_i(k+\Delta) - <\tau_i>)$$

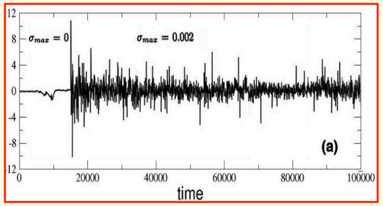
for the previous interoccurrence time series (L is the length of the time series, *th* stands for 'threshold' and A' is a normalization factor). For the three values of threshold considered, we found a **power-law decay** $C_{th}(\Delta) \sim \Delta^{-\gamma(th)}$ with values for the exponent $\gamma(th)$ decreasing with the increase of the threshold and included in the interval [0.34,0.42], again in agreement with analogous results found in financial data. This shows also the **presence of memory effects induced by noise**, in addition to the correlations already pointed out by the deviations from Gaussian behavior quantified by the entropic index q.



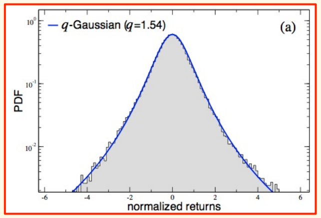
Summary



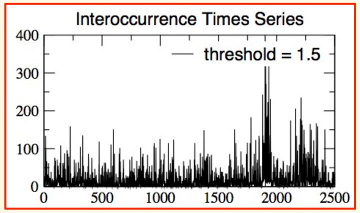
We studied the effect of a weak random **additive noise** in a linear chain of N locally-coupled **logistic maps at the edge of chaos**.



Maps tend to synchronize for a strong enough coupling, but if a weak noise is added, very **intermittent fluctuations** in the returns time series are observed. This intermittency tends to disappear when noise is increased.



From the returns analysis we observe the emergence of **fat tails** which can be satisfactorily reproduced in the context of nonextensive statistical mechanics by ***q*-Gaussians curves.**



Inter-occurrence times of these extreme events show similarities with recent analysis of **financial data**.

SEE ALSO:

- A.Pluchino, A.Rapisarda, C.Tsallis (2012) arXiv:1206.2152v1 [cond-mat]
- A.Pluchino, A.Rapisarda, C.Tsallis, Europhysics Letters 80 (2007) 26002
- G.Miritello, A.Pluchino, A.Rapisarda, Physica A 388 (2009) 4818-4826
- G.Miritello, A.Pluchino, A.Rapisarda, Europhysics Letters 85 (2009) 10007
- J. Ludescher, C. Tsallis and A. Bunde (2011) Europhys. Letters 95, 68002
- C. Li and J. Fang, IEEE 0-7803-8834-8/05 (2005) 288 - 291 Vol. 1
- U.Tirnakli, C.tsallis and C.Beck (2009) Phys. Rev. E 79, 056209 (R)
- K. Kaneko, "*Simulating Physics with Coupled Map Lattices*" (1990) World Scientific, Singapore